Birla Institute of Technology & Science, Pilani

First Semester 2017-2018, MATH F111 (Mathematics I)
Comprehensive Examination

Time: 3 Hours **Date: December 09, 2017 (Saturday)** **Max. Marks: 135**

1. The question paper consists of two parts. **Part A (Closed Book)** is of 75 Marks & 100 Minutes and **Part B (Open Book)** is of 60 Marks & 80 Minutes. Attempt questions of **Part A** and **Part B** on two separate answer sheets.
2. Answer sheet and question paper of **Part B** shall be given only after the submission of **Part A** answer sheet. Late submission of **Part A** is not allowed. On the top right corner of the first answer sheet, write **Part A**, and on the second answer sheet, write **Part B**.
3. Begin solution of each question on a new page, and answer the parts (if any) of each question in continuation. **Calculator is not allowed**. Write **END** at the end of the last attempted solution in each answer sheet. Use ball point pen only for writing the solutions and drawing the diagrams.

Part A (Closed Book) Time: 2:00PM-3:40PM (100 Min.) Max. Marks: 75

A1. (a) Discuss the convergence of the series $\sum_{n=0}^{\infty}(e^x-4)^n$, and for what values of x the series converges absolutely/conditionally. Also find its sum. **[6]**

(b) Trace the polar curve $r = 2(1+\cos\theta)$ by faster graphing method and find its area. **[9]**

A2. (a) Identify and sketch the conic $r = \dfrac{25}{10-5\cos\theta}$ with justification. Also label all its vertices and foci with appropriate polar coordinates. **[9]**

(b) Find the unit tangent vector, principal unit normal and curvature for the space curve $\overrightarrow{r}(t) = (\cos t + t\sin t)\widehat{i} + (\sin t - t\cos t)\widehat{j} + 3\widehat{k}$. **[6]**

A3. Use the limit definition to find the derivative of $f(x,y) = x^2 + xy$ at point $P(1,2)$ in the direction of the vector $\widehat{i}+\widehat{j}$. In what directions does the function f change most rapidly and what are the rates of change in these directions? Also, find the unit normal vector to the plane $z = L(x,y)$, where $L(x,y)$ is linearization of the function $f(x,y) = x^2+xy$ at the point $P(1,2)$. **[6+4+5]**

A4. Sketch the region R bounded by the parabola $x = y^2$ and the line $x - y = 2$, which lies outside the circle $x^2+y^2 = 2$. Find area of the region R by using $\iint_R dx\,dy$. Also, change the order of the integral $\iint_R dx\,dy$ (mention the limits only). **[3+6+6]**

A5. (a) Sketch the region bounded by the planes $4x+2y+z = 10, y = 0, z = 0$ and $x = 0$, and find its volume by using triple integral. **[7]**

(b) Convert the following integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy$$

to an equivalent integral in cylindrical coordinates, and evaluate it. **[8]**

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Note: For the open book examination, the text book (Thomas Calculus, 13th Edition) and hand written class notes are allowed. Any other printed or photocopy material is not allowed. Loose sheets of hand written notes are also not allowed.

Part B (Open Book) **Time: 3:40PM-5:00PM (80 Min.)** **Max. Marks: 60**

B1. Consider the function

$$f(x,y) = \begin{cases} x^2\tan^{-1}\left(\dfrac{y}{x}\right) + y^2\tan^{-1}\left(\dfrac{x}{y}\right), & xy \neq 0 \\ 0, & xy = 0. \end{cases}$$

Find $f_{xy}(0,0)$. Examine the continuity of f at $(0,0)$. Without using polar form, examine the continuity of f_y at $(0,0)$. **[15]**

B2. (a) If A, B and C are the angles of a triangle, locate and classify the critical points of the function $f(A,B,C) = \cos A \cos B \cos C$. (Do not use the method of Lagrange multipliers.) **[7]**

(b) Prove that the equation $\dfrac{l^2a^4}{1-a^2u} + \dfrac{m^2b^4}{1-b^2u} + \dfrac{n^2c^4}{1-c^2u} = 0$ is satisfied by the extreme values of $u = \dfrac{x^2}{a^4} + \dfrac{y^2}{b^4} + \dfrac{z^2}{c^4}$ subject to the constraints $lx + my + nz = 0$ and $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$. Here a, b, c, l, m, n are real constants. **[8]**

B3. (a) Find all $(a,b) \in \mathbb{R}^2$ such that the vector field

$$\overrightarrow{F}(x,y,z) = \ln(1+y^2+z^2)\,\widehat{i} + \frac{(b-a^2)xy}{1+y^2+z^2}\,\widehat{j} + \frac{axz}{1+y^2+z^2}\,\widehat{k}$$

is conservative in $\mathbb{R}^3$, and hence find a potential function for $\overrightarrow{F}$. **[8]**

(b) Set up (do not evaluate) the iterated double integral in polar coordinates for the counterclockwise circulation of the vector field

$$\overrightarrow{F}(x,y) = (\sqrt{1+x^2} - ye^{xy} + 3y)\,\widehat{i} + [x^2 - xe^{xy} + \ln(1+y^4)]\,\widehat{j}$$

along the boundary of the region shared by the circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$. **[7]**

B4. (a) Use Stokes' theorem to evaluate the flux of curl $\overrightarrow{F}$ across a surface S for the vector field $\overrightarrow{F} = (z^2-1)\widehat{i} + (z+xy^3)\widehat{j} + 6\widehat{k}$, where S is the bounded surface of $x = 6-4y^2-4z^2$ cut off by the plane $x=-2$ with orientation in the negative x-direction. **[8]**

(b) Use divergence theorem to evaluate outward flux of the vector field $\overrightarrow{F} = yx^2\widehat{i} + (xy^2-3z^4)\widehat{j} + (x^3+y^2)\widehat{k}$ across the surface S of the solid given by $x^2+y^2+z^2 \leq 16$, $z \leq 0$ and $y \leq 0$. **[7]**

****END****