Birla Institute of Technology & Science, Pilani

First Semester 2017-2018, MATH F111 (Mathematics I)

Mid Semester Examination (Closed Book)

Time: 90 Min. **Date: October 12, 2017 (Thursday)** **Max. Marks: 105**

1. Write solution of each question on fresh page. Answer each subpart of a question in continuation.
2. Write **END** in the answer sheet just after the final attempted solution.

1. Test the absolute and conditional convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n - \ln n}$. **[17]**

2. (a) Find the center, radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (4x-8)^n}{n}$. Also identify the values of x for which the series converges (i) absolutely (ii) conditionally. **[12]**

   (b) Find the first three non-zero terms in the Taylor series expnasion of $\sin x$ about $x = \frac{\pi}{2}$. **[5]**

3. (a) Shade the region in the first quadrant inside the circle $r = \sin\theta$ and outside the curve $r = \cos 2\theta$. Find all the intersection points and label them. Also find area of the shaded region. **[12]**

   (b) Find the length of the curve $r = \sqrt{1 + \sin 2\theta},\ 0 \le \theta \le \pi\sqrt{2}$. **[5]**

4. Find the unit tangent, normal, and binormal vectors at the point $(\sqrt{2}, \sqrt{2}, \frac{\pi}{2})$ of the curve $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + 2t\mathbf{k}$. Also find the curvature at the given point. **[18]**

5. (a) Find and sketch the domain of the function

$$f(x,y) = \frac{\sqrt{x^2 + y^2 - 9}}{x}.$$

   Determine if the domain is an open region, a closed region, or neither. Justify your answer. **[8]**

   (b) Examine the continuity of the following function at $(0,0)$: **[10]**

$$f(x,y) = \begin{cases} \frac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)}, & (x,y) \ne (0,0) \\ \frac{1}{2}, & (x,y) = (0,0) \end{cases}$$

6. (a) Let $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$. Show that $f_x(0,0)$ and $f_y(0,0)$ exist. Is f differentiable at $(0,0)$? Justify. **[9]**

   (b) Let $f(x,y) = x^2 + xy + y^2$ where $x = uv$ and $y = u/v$. Show that $uf_u + vf_v = 2xf_x$. **[9]**

END