BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K.K. BIRLA GOA CAMPUS

FIRST SEMESTER 2019-2020

Comprehensive Eaxamination (Closed Book)

MATH F 111 **MATHEMATICS-I**

Date: December 05, 2019 Time: 2hrs.

Day: Thursday Max. Marks:90

INSTRUCTIONS

1. All questions are compulsory. 2. Begin answering a new question on a fresh page. 3. Write all the steps clearly for complete credit. 4. Number all the pages of your answer book and **make a question-page index** on the front page. Write your **tutorial section/Instructor's name** correctly. A penalty of **5 marks** will be imposed, in case of incomplete the index and wrong section number/instructor's name.

1. Show that the curvature of a smooth plane curve $x = x(t),\ y = y(t)$ is

$$\kappa(t) = \frac{|x^{'}y^{''} - y^{'}x^{''}|}{(x^{'2} + y^{'2})^{3/2}}.$$

Use this formula to show that curvature of the polar curve $r = f(\theta)$ is

$$\kappa(\theta) = \frac{|r^2 + 2(r^{'2}) - rr^{''}|}{[r^2 + r^{'2}]^{3/2}}.$$

[15]

2. Using method of Lagrange multipliers find the point on the plane $x + y - z = 1$ that is closest to the point $(0, -3, 2)$ and calculate the distance also. [10]

3. Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = \sqrt{2}\cos x \sin(y + z)$ at the point $P_0(0, 0, \pi/4)$. Then find an upper bound for the magnitude of the error E in the approximation $f(x, y, z) \sim L(x, y, z)$ over the region R which is defined by $|x| \leq 0.01,\ |y| \leq 0.01$ and $|z| \leq 0.01$. [10]

4. Write the triple integral for finding the volume of the region enclosed in between $z^2 = x^2 + y^2,\ z^2 = 2x^2 + 2y^2,\ z = 1\ \&\ z = 2$ in rectangular coordinates in order $dzdydx$ **(substraction of volume is not allowed)**. [20]

5. Verify Divergence Theorem for the vector field $\mathbf{F} = 2x\mathbf{i} - yz\mathbf{j} + z^2\mathbf{k}$ where the surface is paraboloid $z = x^2 + y^2$ capped by the disk $x^2 + y^2 \leq 1$ in the plane $z = 1$. [20]

6. Show that for the vector field $\mathbf{F} = (xz + \cos^3 y)\mathbf{i} + (yz + \cos^3 x)\mathbf{j} + (\frac{x^2+y^2}{2})\mathbf{k}$, the line integrals

$$\int_{C_1} \mathbf{F}.d\mathbf{r} = \int_{C_2} \mathbf{F}.d\mathbf{r},$$

where C_1 is the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$ oriented counter clockwise and C_2 is the circle $x^2 + y^2 = 1$ on the xy-plane. [15]

****** **Best of Luck** ******