BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K.K. BIRLA GOA CAMPUS

FIRST SEMESTER 2019-2020

Mid-Term Test (Closed Book)

MATH F 111 **MATHEMATICS-I**

Date: October 03, 2019 Time: 90 minutes.

Day: Thursday (11:00–12:30) Max Marks:90

INSTRUCTIONS

1. All questions are compulsory. 2. Begin answering a new question on a fresh page. 3. Write all the steps clearly for complete credit. 4. Number all the pages of your answer book and **make a question-page index** on the front page. Write your **tutorial section/Instructor's name** correctly. A penalty of **5 marks** will be imposed, in case of incomplete the index and wrong section number/instructor's name.

1. Discuss the convergence of the following series: [5+5+5]

   (a) $\sum_{n=1}^{\infty} \frac{1}{n} \tan\left(\frac{1+n^2}{n^2}\right)$

   (b) $\sum_{n=1}^{\infty} \frac{\sqrt{x^2+n}-|x|}{n^2}$, where x any real number.

   (c) $\sum_{n=1}^{\infty} n^n x^n$, where x any real number.

2. Find the interval of convergence and radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n(n+1)}$. Justify your answer. [10]

3. Use $\epsilon - \delta$ definition to prove that the function $f(x,y) = \frac{x^2+y^4+x}{(x-1)^2+y^4+2x}$ is continuous at $(0,0)$. [7]

4. Consider the function

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4+y^2} & \text{if } (x,y) \neq (0,0); \\ 0 & \text{else }. \end{cases}$$

   Find $f_x(0,0)$, $f_y(0,0)$, $f_{xy}(0,0)$ and $f_{yx}(0,0)$, if they exist. Justify your answer. [5+10]

5. Find an equation for the circle of curvature of the curve $\mathbf{r}(t) = (2\ln t)\mathbf{i} - (t + \frac{1}{t})\mathbf{j}$, where $e^{-2} \le t \le e^2$ at the point $(0,-2)$, where $t = 1$. [10]

6. Compute curvature and torsion for the curve $\mathbf{r}(t) = (2\cos t)\mathbf{i} + t\mathbf{j} + (2\sin t)\mathbf{k}$ at $t = \pi$. [5+5]

7. Find the equations of osculating, normal and rectifying planes for the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at the point $(1,1,1)$ [10]

8. Find the area of the common region bounded by both the curves $r = 2 - 3\cos\theta$ and $r = \cos\theta$. [13]

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