Birla Institute of Technology & Science, Pilani

First Semester 2021-2022, MATH F111 (Mathematics-I)

Mid Semester Examination- New Time table (Closed Book)

Time: 90 Min. **Date: March 10, 2022** **Max. Marks: 90**

1. Draw figures as and when required. Notations/symbols have their usual meaning.
2. Start new question on a fresh page. Moreover, answer each subpart of a question in continuation.

1. **(i).** The area enclosed by the curve $r = a + a\cos\theta, a > 0$ is $\frac{147\pi}{2}\cdot$ Find the value of a. **[7]**

(ii). The curve C has polar equation $r = 1 + 2\cos\theta,\ \ 0 < \theta < \pi/2$. At the point P on C, the tangent to C is parallel to the initial ray. Given that O is the pole, find the length of the line OP. **[5]**

2. **(i).** Consider the curve $r^2 = -\sin 2\theta$. **[10]**

(a) Test the curve for symmetries along x-axis, y-axis and origin.

(b) Find the points on the curve where slope of the tangent is $-1, 0$ and ∞.

(c) Using (a) and (b), sketch the curve in polar coordinates.

(ii). Find points of intersection of the cardioid $r = 1 + \cos\theta$ and the circle $r = \sqrt{3}\sin\theta$. Also, find the length of the portion of the cardioid enclosed by the circle. **[8]**

3. **(i).** A particle travelling in a straight line is located at the point $(1, -1, 2)$ and has speed 2 at time $t = 0$. The particle moves towards the point $(3, 0, 3)$ with constant acceleration $2\mathbf{i} + \mathbf{j} + \mathbf{k}$. Find its position vector $\mathbf{r}(t)$ at time t. **[7]**

(ii). Find the point on the curve $\mathbf{r}(t) = (12\sin t)\mathbf{i} - (12\cos t)\mathbf{j} + 5t\mathbf{k}$ at a distance 13π units along the curve from the point $(0, -12, 0)$ in the direction of increasing arc length. Moreover, find torsion τ for the curve $\mathbf{r}(t)$. **[10]**

(iii). For a twice differentiable plane curve $y = f(x)$, derive an expression for the curvature κ in terms of f, f', f''. **[8]**

4. **(i).** Consider the function $f(x, y) = \dfrac{1}{\ln(x^2 + y^2 - 1)}\cdot$ With justification, answer the following:

(a) What is domain D of f? **[3]**

(b) Find boundary of D. **[3]**

(c) Is D open? **[3]**

(d) Identify the level curves of f at a level $0 \neq c \in \mathbb{R}$. **[2]**

(ii). Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \dfrac{x^2 y + x^9}{x^8 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0). \end{cases}$$

(a) Is f differentiable at $(0, 0)$? **[4]**

(b) Find the directional derivative of f at $(0, 0)$ in the direction of a unit vector $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$, if it exists. **[6]**

(iii). Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function. If $z = e^u f(v)$, $u = ax + by$ and $v = ax - by$, then find $g(a, b)$ such that **[8]**

$$b^2 z_{xx} - a^2 z_{yy} = g(a,b)e^u f'(v).$$

5. Let $P(a, b, c)$ be a point on the surface $S : z^2 = x^3 + y^2$, where $a, b, c \in \mathbb{R}$. For what values of a, b and c, does the tangent plane to S at P passes through the origin? **[6]**

END