Birla Institute of Technology & Science, Pilani

MATH F111 (Mathematics-I)
First Semester 2023-2024
Mid-Sem Examination (Closed Book)

Time: 90 Minutes **Date: October 13, 2023 (Friday)** **Max. Marks: 105**

1. Notations and symbols have their usual meaning.
2. Start new question on fresh page. **Moreover, answer each subpart of a question in continuation.**
3. Write **END** at the end of the last attempted question.
4. The use of calculators is **not** permitted.

Q.1 (a) Find the area of the region inside the circle $r = 6\cos\theta$ and outside the cardioid $r = 2(1+\cos\theta)$. **[15]**

(b) Find the length of the portion of the cardioid $r = 2(1+\cos\theta)$ which lies inside the circle $r = 6\cos\theta$. **[11]**

Q.2 (a) Using a suitable parametrization of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b > 0$, find its curvature at a point (x, y) in terms of the defined parameter. Determine the point(s) where the curvature of the ellipse is largest and smallest. **[16]**

(b) Find the torsion for the curve $\mathbf{r}(t) = (\cosh t)\mathbf{i} - (\sinh t)\mathbf{j} + t\,\mathbf{k}$. **[10]**

Q.3 (a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} (x^2+y^2)\ln(x^2+y^2), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

(i) Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$. **[12]**

(ii) Examine the continuity of f_{xy} at $(0,0)$. **[5]**

(b) Consider $z = \ln(f(w))$, $w = g(x,y)$, $x = \sqrt{r-s}$, and $y = r^2 s$. Given the following information: $g_x(2,-9) = -8$, $g_y(2,-9) = 2$, $f'(-2) = 2$, $f(-2) = 4$, and $g(2,-9) = -2$, determine $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial s}$ when $r = 3$ and $s = -1$. **[10]**

Q.4 (a) If $u = 1/r$, where $r = \sqrt{x^2+y^2+z^2}$; and $r \neq 0$, prove or disprove $u_{xx} + u_{yy} + u_{zz} = 0$. **[10]**

(b) Using the method of Lagrangian multipliers find the extreme value(s) of $f(x,y,z) = x^2yz + 1$ on the intersection of the plane $z = 1$ and the sphere $x^2+y^2+z^2 = 10$. **[16]**

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