Birla Institute of Technology & Science, Pilani

Second Semester 2017-2018, MATH F113 (Probability & Statistics)

Comprehensive Examination

Time: 90 Min. **Date: May 8, 2018 (Tuesday)** **Max. Marks: 70**

1. The question paper has two parts, Part A (Closed Book) and Part B (Open Book). Write Part A and Part B on top right corner of the cover page of the respective answer sheets as well as respective supplements.
2. Write solution of each question on a fresh page. Moreover, answer each subpart of a question in continuation.
3. Answer each question legibly, clearly and concisely. Illegible answers will not be graded.
4. For each question, box your final answer(s)/conclusion(s). Write **END** in the answer sheet just after the final solution.

Part A (Closed Book)

1. (a) A diagnostic test has a probability 0.95 of giving a positive result when applied to a person suffering from a certain disease, and a probability 0.10 of giving a positive when applied to a non-sufferer. It is estimated that 0.5% of the population are sufferers. Suppose that the test is applied to a person about whom we have no relevant information relating to the disease (apart from the fact that he/she comes from this population). Determine:
(i) probability that the test result will be positive,
(ii) probability that given a negative result, the person is a non-sufferer,
(iii) probability that the person will be diagnosed wrongly.

   (b) For a die, the probability of the face with j dots is proportional to j for $j = 1, 2, \ldots, 6$. Find the probability that a face with odd number of dots will turn up in one roll of the die. **[8+6]**

2. A pair of unbiased six-sided dice is rolled independently.

   (a) Let X denote the smaller value of the outcomes if they are different and the common value if they are equal. Find the probability density function of X and hence the expected value of X.

   (b) Let Y denote the absolute value of the difference of the outcomes. Determine the probability density function of Y. Also find the expected value of Y^2. **[7+7]**

3. (a) Suppose that an average of 30 customers per hour arrive at a shop in accordance with a Poisson process. Find the probability that
(i) at most two customers arrive during a 6 minute period,
(ii) the shopkeeper will wait between 3 to 5 minutes for the arrival of the first customer.

   (b) Buses arrive at a specified stop at 15 minute intervals starting at 7:00 AM. If a passenger arrives at the stop at a time that is uniformly distributed between 7:00 AM and 7:30 AM, find the probability that he waits
(i) less than 5 minutes for a bus, (ii) at least 12 minutes for a bus. **[7+7]**

4. For the random variables X and Y, consider the function

$$f(x,y) = \begin{cases} Kxy, & x > 0,\, y > 0,\, x + y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(i) Find the value of the constant K which makes f the joint density of X and Y. Hence find (ii) the conditional density function of X given $Y = y$, (iii) $\mu_{X|y}$ and (iv) $P[Y \geq X]$. **[14]**

5. (a) The experiments in the Chemistry lab yield the following data concerning the temperature X in degree Celsius and the solubility Y of Sodium Nitrate in 10 parts of water.

x	0	4	10	15	20
y	6	7	7	8	8

Using normal equations, estimate the linear regression equation of Y on x. Hence estimate the average solubility of Sodium Nitrate in 10 parts of water when the temperature is 30^0 Celsius.

(b) If a sample of size 12 on (X, Y) gives $\sum x = 84, \sum y = 56, \sum x^2 = 672, \sum y^2 = 308$ and $\sum xy = 452$, find the sample correlation coefficient r of this sample. Based on this sample, will you support the guess that small values of Y tend to be associated with small values of X? Justify. [8+6]

END