Birla Institute of Technology & Science, Pilani

Second Semester 2017-2018, MATH F113 (Probability & Statistics)
Mid Semester Examination (Closed Book)

Time: 90 Min. **Date: March 8, 2018 (Thursday)** **Max. Marks: 105**

1. Write solution of each question on a fresh page. Moreover, answer each subpart of a question in continuation.
2. Answer each question legibly, clearly and concisely. Illegible answers will not be graded.
3. For each question, box your final answer(s)/conclusion(s). Write **END** in the answer sheet just after the final solution.

1. Each Sunday a statistician visits one of the three possible locations near his home for fishing: he goes to the sea with probability 0.50, to a river with probability 0.25, or to a lake with probability 0.25. If he goes to the sea there is an 80% chance that he will catch fish; corresponding figures for the river and the lake are 40% and 60%, respectively.
   (a) Find the probability that, on a given Sunday, he catches fish. **[7]**
   (b) If, on a particular Sunday, he comes home without catching anything, where has he most likely been? **[10]**

2. If a random variable X has the following probability density function

$$f(x) = \begin{cases} k(x-3)^2 & ; \quad x = 3, 4, 5 \\ 0 & ; \quad \text{elsewhere.} \end{cases}$$

   (a) Find the value of k and hence compute the moment generating function of X. **[8]**
   (b) Using moment generating function of X, find its mean and variance. **[10]**

3. (a) Suppose that in flight, airplane engines will fail with probability $(1-p)$ independently from engine to engine. If an airplane needs a majority of its engines operative to make a successful flight, for what values of p is a 5-engine plane preferable to a 3-engine plane? **[10]**
   (b) Derive the moment generating function of the Poisson distribution with parameter k. **[8]**

4. The diameter of an electric cable X is a continuous random variable with probability density function

$$f(x) = \begin{cases} kx(1-x) & ; \quad 0 \le x < 1 \\ 0 & ; \quad \text{elsewhere.} \end{cases}$$

   (a) Find the value of k. Hence, **[3]**
   (b) find $E\left[e^X\right]$, and **[6]**
   (c) calculate $P(X \le \frac{1}{2} | \frac{1}{3} < X < \frac{2}{3})$. **[8]**

5. Let X be a gamma random variable with parameters α and β. Find the moment generating function of X and hence compute the mean and variance of X. **[18]**

6. A company has installed 10000 electric bulbs in a metro city. Given that 10% of the bulbs are likely to fail after 744 hours of burning while 10% of the bulbs are likely to survive after 1256 hours of burning. Assuming normality of the lifetime of the bulbs, how many bulbs are expected to burn between 800 and 1200 hours. **[17]**

Cumulative Standard Normal Distribution:
$F(-1) = 0.1587$, $F(-1.28) = 0.1$, $F(1.42) = 0.9222$, $F(2.31) = 0.9896$

END