

Birla Institute of Technology & Science, Pilani (Raj.)

Second Semester 2022-2023

MATH F113 (Probability & Statistics)
Mid-Semester Examination (Closed Book)

Time: 90 Minutes

Date: May 01, 2023 (Monday)

Max. Marks: 105

1. There are total 3 questions. Start answering each question on a new page. Answer each sub-part of the questions in continuation. While answering, write all the steps with proper justification. Just writing the correct answer will receive no credit.
2. Define events/random variables as and when required.
3. Notations/symbols have their usual meaning.

- Q 1.** (a) Let $n > 2$. Bag A contains n red and 2 white balls. Bag B contains 2 red and n white balls. Bag C contains $n - 2$ red and 4 white balls. One of the three bags is selected at random and two balls are drawn from it without replacement. Suppose that the probability that Bag A was used to draw the balls given that both the balls drawn are red equals $\frac{3}{4}$. Find the possible values of n . [15]
- (b) A bag contains three red balls, four blue balls, and five green balls. Suppose that three balls are randomly selected. What is the probability that all three of the selected balls have the same color? What is the probability that atleast two of the selected balls are green? What is the probability that one ball of each color is selected? [15]
- Q 2.** (a) A biased die is such that the probability of getting a six is less than $\frac{1}{6}$. An experiment consists of recording the number of sixes in 25 throws of the die. If the standard deviation of the number of sixes is $\frac{3}{2}$, then find the probability that exactly three sixes are recorded during a particular experiment. [13]
- (b) An oil company conducts a study that indicates that an exploratory oil well should have a 10% chance of striking oil. What is the probability that the first strike comes on the sixth well drilled? [7]
- (c) Let X be a random variable with the moment generating function

$$m_X(t) = \frac{1}{8}e^{-t} + \frac{1}{4}e^t + \frac{1}{4}e^{2t} + \frac{1}{4}e^{4t} + \frac{1}{8}, \quad t \in \mathbb{R}.$$

Then find $V(X)$ and $P(X \geq 2)$. [8+7]

- Q 3.** (a) Let X be a Poisson distributed random variable with parameter μ . Using the definition of expectation, derive $E[X^2]$. [10]
- (b) The total allocated time of 30 minutes for a sitcom is divided into content time and the commercial time. The content time in minutes is uniformly distributed on the interval $[18, 26]$. Calculate the probability that the commercial time will be more than 10 minutes. [8]
- (c) Let X be a continuous random variable with pdf given by

$$f(x) = \frac{1}{2}e^{-|x|}, \quad x \in \mathbb{R}.$$

If $Y = X^2$, find the cdf of Y . [10]

Please turn over...

- (d) The time needed to complete a final examination in a particular college course is normally distributed with a mean of 80 minutes and a variance of 100. Assume that the class has 60 students and that the examination period is 90 minutes in length. What is the probability that a student will not be able to complete the exam in the allotted time? Find the expected number of students in the class who will be able to complete the exam in the allotted time when the performance of students are independent. (Given that $P(0 \leq Z \leq 1) = 0.3413$, $P(0 \leq Z \leq 0.1) = 0.0398$) **[12]**

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