Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2016-2017, MATH F211 (Mathematics III)

Comprehensive Examination

Time: 3 Hours **Date: December 03, 2016 (Saturday)** **Max. Marks: 120**

1. The question paper consists of two parts. **Part A (Closed Book)** is of 40 Marks & 60 Minutes and **Part B (Open Book)** is of 80 Marks & 120 Minutes. Attempt questions of **Part A** and **Part B** on two separate answer sheets.
2. Answer sheet of **Part B** shall be given only after the submission of **Part A** answer sheet. Late submission of **Part A** is not allowed.
3. On the top right corner of the first answer sheet, write **Part A**, and on the second answer sheet, write **Part B**.
4. Begin solution of each question on a new page, and answer the parts (in any) of each question in continuation. **Calculator is not allowed**.
5. Write **END** at the end of the last attempted solution in each answer sheet.

Part A (Closed Book)

Time: 1 Hour **Max. Marks: 40**

Q. 1 (a) Apply the method of variation of parameters to find a particular solution of the differential equation

$$x^2y'' + 3xy' + y = \frac{1}{(1-x)^2},$$

and hence write its general solution. **[12]**

(b) Solve the following differential equation:

$$y'(x^2y^3 + xy) = 1.$$ **[8]**

Q. 2 (a) Find Fourier series of the function $f(x) = |\sin x|, \quad -\pi \le x < \pi.$ **[5]**

(b) Using the method of separation of variables, derive a solution for the following boundary value problem:

$$\begin{aligned} y_{tt} &= a^2 y_{xx}; \quad 0 < x < L, \quad t > 0 \\ y(0,t) &= y_x(L,t) = 0; \quad t \ge 0 \\ y(x,0) &= f(x), \quad y_t(x,0) = g(x); \quad 0 < x < L \end{aligned}$$ **[15]**

END Part A (Closed Book)

Part B (Open Book)

Time: 2 Hours **Max. Marks: 80**

Q. 1 Find the general solution of the differential equation

$$y'' - 4(\sec x + \tan x)y' - 4(y-2) = 0.$$

about $x = \pi/2$ in terms of hypergeometric functions. **[20]**

Q. 2 (a) Show that

$$\frac{x}{2} J_{p-1}(x) = \sum_{k=0}^{\infty} (-1)^k (p+2k) J_{p+2k}(x).$$ **[8]**

(b) Prove that

$$\sum_{n=0}^{\infty} \frac{P_n(\cos\theta)}{n+1} = \log\left[\frac{\sin(\theta/2)+1}{\sin(\theta/2)}\right].$$ **[12]**

Q. 3 (a) Find a closed form expression for the Laplace transform $L[f(x)]$ of $f(x) = e^{\{x\}}$, where $\{x\} = x - [x]$ denotes the fractional part of x. Hence, find a function $h(x)$ such that $L^{-1}\left[\frac{e^p - e}{(e^p-1)(p^3-1)}\right]$ is the convolution of $f(x)$ and $h(x)$. **[10]**

(b) Show that in any interval of length π on the positive X-axis, any nontrivial solution of

$$\left(e^{2x}-1\right) y'' + 2\left(e^{2x}+1\right) y' + (e^{2x}-1)(e^x+3)y = 0$$

has at least one zero. **[10]**

Q. 4 (a) Reduce the differential equation

$$y''' + 3x^2(y'')^2 - \sin(y') + (y')^3 + 3xy = 0$$

to a system of first order differential equations. **[5]**

(b) Find all the eigen values for the following boundary value problem:

$$y'' + y' + \lambda y = 0; \quad y'(0) = 0, \quad y'(\pi) = 0,$$

where λ is a real number. **[15]**

END Part B (Open Book)