Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2016-2017, MATH F211 (Mathematics III)

Mid Semester Examination (Closed Book)

Time: 90 Min. **Date: October 04, 2016 (Tuesday)** **Max. Marks: 90**

1. Write solution of each question on fresh page. Answer each subpart of a question in continuation.
2. Write **END** in the answer sheet just after the last attempted solution.

1. (a) Test the exactness of the differential equation

$$(e^y + 2x)dx + (xe^y + 2y)dy = 0.$$

If the above given equation is exact, find its solution. If it is not exact, then make it exact by finding an integrating factor, and hence solve it. **[10]**

(b) Let $y_1(x)$ be a given non-trivial solution of the differential equation

$$y'' - h(x)y = 0,$$

where $h(x)$ is continuous on $[a, b]$. Then find the second linearly independent solution $y_2(x)$ of the above given differential equation, writing all the steps in the derivation of $y_2(x)$. Find the Wronskian of $y_1(x)$ and the derived solution $y_2(x)$. **[10]**

(c) Consider the differential equation

$$y'' + P(x)y' + Q(x)y = 0,$$

where $P(x)$ and $Q(x)$ are continuous on $[a, b]$. If $y_1(x)$ and $y_2(x)$ are any two solutions of the above equation on [a, b], then show that their Wronskian $W(y_1, y_2)$ is either identically zero or never zero on [a, b]. **[10]**

2. (a) Transform the differential equation

$$(1 + x^2)^2 y'' + 2(x - 2)(1 + x^2)y' + 4y = 0,$$

into a differential equation with constant coefficients by using an appropriate transformation, and hence solve it. **[12]**

(b) Find the general solution of the differential equation

$$y'' - 7y' + 12y = e^{2x}(x^3 - 5x^2),$$

with the help of the operator method. **[12]**

(c) Find the normal form of the differential equation $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0$, where n is a non-negative integer and $|x| < 1$. **[6]**

3. (a) Show that $x = 0$ is a regular singular point of the differential equation $xy'' + 2y' + xy = 0$. Find its Frobenius series solution corresponding to the larger root of the indicial equation around $x = 0$, and express it in terms of elementary functions. **[18]**

(b) Transform the differential equation $(1 - x^2)y'' - xy' + y = 0$ into a hypergeometric differential equation, and hence express its general solution near $x = -1$ in terms of hypergeometric functions. **[12]**

END