Birla Institute of Technology and Science, Pilani

First Semester, 2017-18

MATH F211 (Mathematics III)

Mid Semester Examination (Closed Book)

Max. Marks: 105 | **Max. Time: 90 Minutes** | **Date: Oct. 09, 2017**

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Note: Start answering each question on a new page. Answer all sub-parts together.

Q1. (a) Find the general solution of the differential equation

$$xy' + 2 = x^3(y-1)y',$$

by converting it into linear differential equation. **[14]**

(b) Solve the differential equation $2yy'' = 1 + (y')^2$. **[12]**

Q2. (a) Find a particular solution of the differential equation

$$y'' - 4y' + 3y = \sin 3x \cos 2x,$$

by using operator method, and hence find the general solution. **[10]**

(b) Find a particular solution of the following differential equation

$$x^2y'' - 3xy' + 4y = \log x \qquad (x > 0),$$

by using method of variation of parameters, and hence find the general solution. **[16]**

Q3. (a) By a suitable change of dependent variable, reduce the differential equation

$$(x-2)(x^2-5x+6)y'' + 4(x^2-5x+6)y' + (x^2-6)y = 0,$$

to a normal form. Hence decide whether a nontrivial solution of this differential equation has infinitely many zeros on negative x-axis. **[13]**

(b) In terms of hypergeometric functions, find the general solution near $x = 2$ of the differential equation

$$(9x^2 - 9x - 18)y'' + (24x - 30)y' + 4y = 0.$$ **[13]**

Q4. (a) Determine whether $x = 0$ is an ordinary point or a regular singular point or an irregular singular point for the following differential equations with proper justification:

(i) $(e^x - 1)y'' + (\sin x)\, y' + xy = 0;$

(ii) $x^3y'' + (\sin x - x)y' + (\sin x)y = 0;$ **[3+3]**

(b) Using the method of Frobenius series solution, find a Frobenius series solution of the differential equation

$$(x^2 + 2x)y'' + 2(1+x)y' - 2y = 0,$$

near $x = 0$. Hence find the general solution. Do not use hypergeometric functions. **[21]**

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