BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K K BIRLA – GOA CAMPUS

MID SEMESTER EXAM (CLOSED BOOK)

FIRST SEMESTER 2019-2020

MATHEMATICS-III | **MATH F211**

Date: October 3, 2019 | Time: 90 Minutes

Day: Thursday | Max. Marks: 90

INSTRUCTIONS: 1. There are 3 questions. All questions are compulsory. 2. Begin answering a new question on a fresh page. 3. Write all the steps clearly and give explanations for complete credit. 4. Number all the pages of your answer book and **make a question-page index** on the front page of main answer sheet. A penalty of **2 marks** will be imposed, in case the index is incomplete. 5. Calculator exchange is not allowed.

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1. **(a)** Use method of undetermined coefficients to find a particular solution of the following differential equation and hence write its general solution [14]

$$x^2y'' - xy' - 3y = x^3 \ln x, \quad x > 0.$$

(b) Show that if $\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) / (Ny - Mx)$ is a function $g(z)$ of the product $z = xy$, then show that $\mu = e^{\int g(z)dz}$ is an integrating factor for the differential equation $M(x, y)dx + N(x, y)dy = 0$. Use it to find the general solution of the following differential equation [16]

$$xdy + ydx + 3x^3y^4dy = 0.$$

2. **(a)** Let $y_1(x) = 1 - x$ and $y_2(x) = x^3$. Show that **(i)** y_1 and y_2 are linearly independent in the interval $[-1, 2]$, and **(ii)** $y(x) = c_1y_1(x) + c_2y_2(x)$ can not be general solution to the differential equation $y''+p(x)y'+q(x)y = 0$, where $p(x)$, $q(x)$ are continuous functions on $[-1, 2]$ and c_1, c_2 are arbitrary constants. [14]

(b) Use method of variation of parameters to find a particular solution and hence write the general solution of the following nonhomogeneous linear system of differential equations [16]

$$\begin{aligned} \frac{dx}{dt} &= -2x + y + 2e^{-t} \\ \frac{dy}{dt} &= x - 2y. \end{aligned}$$

3. **(a)** Use operator method to find a particular solution of the following differential equation and hence write the general solution [14]

$$y''' - 4y'' + 5y' - 2y = e^x \sin x.$$

(b) Find two linearly independent Frobenius series solutions of the following differential equation near the singular point $x = 0$. Hence, write its general solution. [16]

$$xy'' + 2y' + xy = 0.$$

Represent solutions in terms of elementary functions.

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