No. of wrong answers	No. of correct answers	Marks Obtained

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K K BIRLA – GOA CAMPUS
FIRST SEMESTER 2019-2020

MATHEMATICS-III **PART-A (Comprehensive Exam, Closed Book)** MATH F211

Date: December 5, 2019 Time: 1 hour Max. Marks: 45

INSTRUCTIONS: **(1)** There are 15 questions. A correct choice for question 1 to 12 will fetch 3 marks and wrong will deduct 1 mark. Question numbers 13 to 15 are of 3 marks. **(2)** Write your name, section number, ID number and room number in the space provided. **(3)** Each question is provided with empty box. You need to write your option/answer with pen within the box in a very clear handwriting. Answers written out of the box will not be considered for evaluation. **(4)** Rough work for Part-A can be done on the back side of main answer sheet. **(5)** Answering with pencil, canceling and overwriting will be considered as invalid. **(6)** Calculator exchange is not allowed.

Name	**ID Number**	**Section No.**	**Room No.**

EXCEPT YOUR ANSWER, DO NOT WRITE OR MARK BELOW THIS LINE

1. The integrating factor for the differential equation $(5xy^2 - 2y)dx + (3x^2y - x)dy = 0$ is
   A. x^3y, B. xy^3,
   C. x^3y^3, D. None of these.

2. The solution of the initial value problem $x^2y'' - xy' - 3y = 0,\ y(1) = 1,\ y'(1) = -2$ is
   A. $\frac{5}{4x} - \frac{x^3}{4}$, B. $\frac{5x}{4} - \frac{x^2}{4}$,
   C. $\frac{3}{4x} - \frac{x^3}{2}$, D. None of these.

3. The values of $\alpha \in \mathbb{R}$ for which all the solutions of $y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0$, tends to zero as $x \to \infty$ are
   A. $\alpha > 0$, B. $\alpha < 0$,
   C. $\alpha = 3$, D. None of these.

4. The general solution of $x^2y'' - xy' + \left(x^2 - 4\right)y = 0$ on $(0, \infty)$, is
   A. $y(x) = c_1J_2(x)$, B. $y(x) = c_1J_{-2}(x)$,
   C. $y(x) = c_1J_2(x) + c_2J_{-2}(x)$, D. None of these.

5. The Legendre polynomial $P_1(x)$ is equal to
   A. $F(-1, 1, 1, \frac{1}{2}(1 - x))$, B. $F(-1, 2, 1, \frac{1}{2}(1 - x))$,
   C. $F(-1, 3, 1, \frac{1}{2}(1 - x))$, D. None of these.

6. Which of the following pair of functions is not a linearly independent solution of the differential equation: $y'' + 9y = 0$?

A. $\{\sin 3x, \sin 3x - \cos 3x\}$,
B. $\{\sin 3x + \cos 3x, 3\sin x - 4\sin^3 x\}$,
C. $\{\sin 3x, \sin 3x \cos 3x\}$,
D. None of these.

7. The inverse Laplace transformation of $F(p) = \tan^{-1}\left(\dfrac{3}{p+2}\right)$ is

A. $\dfrac{e^{-2x}\sin 3x}{x}$,
B. $\dfrac{e^{-2x}\cos 3x}{x}$,
C. $\dfrac{e^{-2x}\tan 3x}{x}$,
D. None of these.

8. Let a and b be positive constants. Then the value of the integral $\displaystyle\int_0^\infty \frac{e^{-ax}\sin bx}{x}dx$ is

A. $\tan^{-1}(a/b)$,
B. $\tan^{-1}(b/a)$,
C. Does not exist,
D. None of these.

9. If $\displaystyle\sum_{n=0}^{\infty} a_n \cos nx$ is the cosine series of $f(x) = \sin x,\ 0 \le x \le \pi$, then the values of a_0 and a_1 are

A. $a_0 = 0 = a_1$,
B. $a_0 = \frac{2}{\pi},\ a_1 = 0$,
C. $a_0 = -\frac{2}{\pi},\ a_1 = 0$,
D. None of these.

10. For $m = n$, the integral $\displaystyle\int_0^1 xJ_p(\lambda_m x)J_p(\lambda_n x)dx$, where λ_n are positive zeros of $J_p(x)$, is equal to

A. $\frac{1}{2}J_{p+1}^2(\lambda_n)$,
B. $\frac{1}{2}J_p^2(\lambda_n)$,
C. 0,
D. None of these.

11. The Laplace transform of $f(x) = xJ_1(x)$ is

A. $-\dfrac{1}{(p^2+1)^{3/2}}$,
B. $\dfrac{1}{(p^2+1)^{3/2}}$,
C. $-\dfrac{p}{(p^2+1)^{3/2}}$,
D. None of these.

12. Fourier series of the periodic function $f(x) = x + x^2, -\pi \le x < \pi$ at $x = \pi$ converges to

A. π,
B. 2π,
C. π^2,
D. None of these.

13. The solution of the integral equation $3\sin(2x) = y(x) + \displaystyle\int_0^x (x-t)y(t)dt$ is

14. The eigenvalues of the BVP: $y'' + \lambda y = 0,\ y'(0) = 0,\ y'(\pi) = 0$ are

15. An equivalent system of first order, for the differential equation $y'' + 2y' - 5y = \sin x$ is

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FIRST SEMESTER 2019-2020
Comprehensive Examination (Closed Book)

MATHEMATICS-III **MATH F211**
Date: December 5, 2019 Time: 2 Hours
Day: Thursday **PART-B** Max. Marks: 85

INSTRUCTIONS

1. There are 3 questions. All questions are compulsory. 2. Begin answering a new question on a fresh page. 3. Questions under each PART must be answered together in continuous pages. 4. Write all the steps clearly and give explanations for complete credit. 5. Number all the pages of your answer book and **make a question-page index** on the front page. A penalty of **2 marks** will be imposed, in case the index is incomplete.

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1. **(a)** Show that [14]

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

where $P_n(x)$ is a Legendre polynomial of degree n and hence find the value of the following integral

$$\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx.$$

(b) Find the Fourier series of the function [14]

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 \le x < \pi \end{cases}$$

and then use it to find the sum of the series

$$\frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \cdots .$$

2. **(a)** Find the general solution of the following differential equation near $x = 3$ in terms of Gauss hypergeometric function [12]

$$(x^2 - x - 6)y'' + (5 + 3x)y' + y = 0.$$

(b) Prove that [16]

$$\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x)\right] = 2\left[\frac{n}{x}J_n^2(x) - \left(\frac{n+1}{x}\right)J_{n+1}^2(x)\right]$$

and then use it to show

$$J_0^2(x) + 2\left(J_1^2(x) + J_2^2(x) + \cdots + J_{99}^2(x)\right) + J_{100}^2(x) = -200 \int \frac{J_{100}^2(x)}{x} dx + C$$

where C is an arbitrary constant.

3. **(a)** Show that [13]

$$L\left(\int_0^x f(t)dt\right) = \frac{F(p)}{p}.$$

Use it to find the solution of the following problem

$$y'' + y' + \int_0^x y'(t)dt = 1,\ y(0) = 0,\ y'(0) = 0.$$

(b) Solve the following heat equation by the method of separation of variables [16]

$$\begin{aligned} u_t &= 3u_{xx} \qquad 0 < x < \pi,\ t > 0 \\ u(0,t) = & \ 0 \ = u(\pi, t),\quad t > 0 \\ u(x,0) &= x,\ \ 0 < x < \pi. \end{aligned}$$

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