

Birla Institute of Technology & Science, Pilani

MATH F211 (Mathematics III)

First Semester 2022-2023

Comprehensive Examination

Time: 180 Minutes

Date: December 17, 2022 (Saturday)

Max. Marks: 135

1. The question paper consists of two parts: **Part–A (Closed Book) for 80 minutes** and **Part–B (Open Book) for 100 minutes**. Attempt questions of Part–A and Part–B in two separate answer sheets.
2. Write Part–A and Part–B on top right corner of the two answer sheets provided. Answer each subpart of a particular question in continuation.
3. Part–B question paper and answer sheet will be given only after submission of part–A answer sheet. Early submission of Part–A is allowed. Write **END** in the answer sheet just after the final solution.

Max Marks: 60

Part-A (Closed Book)

Time: 80 Minutes

Q.1 (a) Find two independent Frobenius series solutions of the equation

[14+6]

$$xy'' - y' + 4x^3y = 0.$$

(b) If $y_1(x)$ and $y_2(x)$ are any two solutions of the equation

$$y'' + P(x)y' + Q(x)y = 0,$$

with P and Q being continuous functions on the interval $[a, b]$, then show that their Wronskian $W = W(y_1, y_2)$ is either identically zero or never zero on $[a, b]$.

Q.2 Using the method of variation of parameters, find the general solution of the following system of differential equations [20]

$$\frac{dx}{dt} = x + y + 3e^{3t}$$

$$\frac{dy}{dt} = 4x - 2y + 4e^{3t}$$

without converting into a second order differential equation.

Q.3 (a) Find the Fourier series for the function

[13+7]

$$f(x) = x \sin x, \quad -\pi < x < \pi,$$

and hence, find the value of

$$1 + 2 \left(\frac{1}{1 \times 3} - \frac{1}{3 \times 5} + \frac{1}{5 \times 7} - \dots \right).$$

(b) Find the Fourier sine series for

$$f(x) = x(\pi - x), \quad 0 < x < \pi.$$

END