Birla Institute of Technology & Science, Pilani
MATH F211 (Mathematics III)
First Semester 2022-2023
Mid-Sem Examination (Closed Book)

Time: 90 Minutes   Date: October 31, 2022 (Monday)   Max. Marks: 105

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1. Notations and symbols have their usual meaning.

2. Start new question on fresh page. **Moreover, answer each subpart of a question in continuation.**

3. Write **END** at the end of the last attempted question.

Q.1 (a) Examine the exactness of the differential equation

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0.$$

Hence, solve it by finding an appropriate integrating factor.   **[10]**

(b) Find a general solution of the differential equation

$$2(x^2 - 5x + 6)y'' + (2x - 3)y' - 8y = 0$$

in terms of Hypergeometric functions near $x = 2$.   **[16]**

Q.2 Without using power series method:
(a) Find the general solution of the differential equation

$$4xy'' + 2y' + 9y = \sin(3\sqrt{x}), \quad x > 0.$$

     [18]

(b) Find two linearly independent solutions of the differential equation

$$y'' - \left(1 + \frac{1}{x}\right)y' + \frac{y}{x} = 0, \quad x > 0,$$

and hence, determine the solution satisfying the conditions $y(1) = 0$ and $y'(1) = 1$.   **[8]**

Q.3 (a) Using the method of variation of parameters, find a particular solution of the differential equation

$$x^2 y'' - 2x(1 + x)y' + 2(1 + x)y = x^3, \quad x > 0.$$

     [15]

(b) Using the Sturm-Comparison theorem, show that every non-trivial solution of the differential equation

$$2x^2 y'' + 2x^3 y' + [(1 + x^2)^2 - (1 + x^3)]y = 0$$

has infinite number of zeros on the positive x-axis.   **[14]**

Q.4 Consider the differential equation
$$x(x - 1)y'' + 3xy' + y = 0.$$

(a) Find all possible ordinary and singular points of the given differential equation and classify its singular points (regular or irregular) with proper justification.   **[6]**

(b) Find all possible Frobenius series solution(s) of the given differential equation near $x = 0$.   **[18]**

END