

Birla Institute of Technology & Science, Pilani

MATH F211 (Mathematics-III)

First Semester 2023-2024

Mid-Sem Examination (Closed Book)

Time: 90 Minutes

Date: October 12, 2023 (Thursday)

Max. Marks: 105

1. Notations and symbols have their usual meaning.
2. Start a new question on a fresh page. **Moreover, answer each subpart of a question in continuation.**
3. Write **END** at the end of the last attempted question.

Q.1 (a) Examine the exactness of the differential equation

$$y(2x^2 - xy + 1)dx + (x - y)dy = 0.$$

If it is exact, then find the general solution. Otherwise, find its general solution by determining an appropriate integrating factor. [10]

(b) Let y_1 and y_2 be two linearly independent solutions of the differential equation

$$y'' + P(x)y' + Q(x)y = 0,$$

where $P(x)$ and $Q(x)$ are continuous functions of x in $[-1, 1]$. Derive expressions for $P(x)$ and $Q(x)$ in terms of y_1, y_2 , and their derivatives. Further, if $y_1 = e^{2x}$ and $y_2 = xe^{2x}$ are the solutions of the given differential equation, then find the value of $P(0) + Q(0) + 2023$. [16]

Q.2 (a) Identify one solution of the differential equation

$$xy'' - 2(x+1)y' + (x+2)y = 0, \quad x > 0.$$

Using this, find another linearly independent solution. Hence, write the general solution of the given differential equation. [10]

(b) Using the method of undetermined coefficients, find a particular solution of the differential equation

$$y'' - 3y' + 2y = e^x(x + \cos x). \quad [16]$$

Q.3 (a) Using the method of variation of parameters, find a particular solution of the differential equation

$$(2x+1)y'' - 2y' - (2x+3)y = (2x+1)^2e^{-x}. \quad [16]$$

(b) Using the Sturm comparison theorem, show that every non-trivial solution of the differential equation

$$(e^{2x} - 1)y'' + 2(e^{2x} + 1)y' + (e^{2x} - 1)(e^x + 3)y = 0$$

has an infinite number of zeros on the positive x -axis. [11]

Q.4 (a) Find the general solution of the differential equation $(x^2 + 1)y'' - 4xy' + 6y = 0$ in terms of power series around $x = 0$. [10]

(b) In terms of hypergeometric functions, find the general solution near $x = -2$ of the differential equation

$$(x^2 - 5x - 14)y'' + (4x + 2)y' + 2y = 0. \quad [16]$$

END