Birla Institute of Technology and Science, Pilani (Raj.)

First Semester, 2023-24

MATH F212 (Optimization)

Mid Semester Examination (Closed Book)

Max. Marks: 70 **Max. Time: 90 Minutes** **Date: OCT. 11, 2023, 11.00 am -12.30 pm**

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Q.1 The manufacturer utilizes three resources: person-hours, machine hours, and cloth material to produce two dress types, Type A and Type B. Type A dresses generate a profit of ₹160 each, while Type B dresses yield a profit of ₹|180 each. The manufacturer's daily capacity allows for the production of either 50 Type A dresses or 20 Type B dresses in terms of person- hours. When it comes to machine hours, the capacity permits the production of 36 Type A dresses or 24 Type B dresses each day. The daily availability of cloth material is limited but sufficient for 30 dresses of either type. Formulate this as a linear programming problem to maximize profit and convert it into standard form. [6]

Q.2 Suppose that the feasible region of the LPP

Max $c^T x$ subject to $Ax = b$, is bounded,

where $A = [a_{ij}], 1 \le i \le m, 1 \le j \le n, c = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}, x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$.

Show that the optimal solution of this LPP always occur at one of the feasible region's extreme point. [10]

Q.3 Solve the following LPP by using the two-phase method:

$$
\begin{aligned}
\text{Maximize } z &= x_1 + 2x_2 + 3x_3 - x_4 \\
\text{subject to } \quad & x_1 + 2x_2 + 3x_3 \ge 15 \\
& 2x_1 + x_2 + 5x_3 = 20 \\
& x_1 + 2x_2 + x_3 + x_4 = 10 \\
& x_1, x_2, x_3, x_4 \ge 0
\end{aligned}
$$

[14]

Q.4 Write the dual of the following primal problem

$$
\begin{aligned}
\text{Min } z &= 5x_1 + 6x_2 \\
\text{subject to } \quad & x_1 + x_2 \ge 2 \\
& 4x_1 + x_2 \ge 4 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

Solve primal problem by dual simplex method. [12]

Q.5 Solve the LPP by Revised Simplex method

$$
\begin{aligned}
\text{Max } z &= x_1 + 2x_2 \\
\text{Subject to } \quad & x_1 + x_2 \le 3 \\
& x_1 + 2x_2 \le 5 \\
& 3x_1 + x_2 \le 6 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

[10]

P.T.O.
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Q.6 The optimal table of the following LP

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

by solving M-method is

Basis	**Z**	x_1	x_2	s_1	R_1	R_2	s_3	**Solution**
z	1	0	0	$-\frac{1}{5}$	$\frac{2}{5} - M$	$\frac{1}{5} - M$	0	$\frac{12}{5}$
x_1	0	1	0	$\frac{1}{5}$	$\frac{3}{5}$	$-\frac{1}{5}$	0	$\frac{3}{5}$
x_2	0	0	1	$-\frac{3}{5}$	$-\frac{4}{5}$	$\frac{3}{5}$	0	$\frac{6}{5}$
s_3	0	0	0	1	1	-1	1	0

Suppose one activity (x_3) is added to the given LPP with coefficients $(2, 1, 1)^T$ for constraints and its coefficient in the objective function is 3. Use post optimal analysis to find the solution of new LPP. [6]

Q.7 Find the initial basic feasible solution for the following transportation problem by VAM.

	D1	D2	D3	D4	
S1	1	5	3	3	32
S2	3	7	1	2	15
S3	0	2	7	3	12
	21	12	11	17	

This solution is degenerate or not. Justify your answer. [12]

****END****