Birla Institute of Technology and Science, Pilani

First Semester, 2023-2024
MATH F212 (Optimization)
Comprehensive exam, PART A (closed book)
Max Marks: 50 December 11, 2023 (Monday) Max time: 90 Mins

Note: 1. Question paper consist 2 parts; Part A (closed book), Part B (open book). Attempt the parts in separate answer book.
2. Notation/symbols have their usual meaning.
3. Start answering each question on new page, and answer the subpart of each question in continuation. Write **END** at the last attempted question.

Q.1 A company has four plants and three warehouses, the supply and demand in units and the corresponding transportation cost per unit is given below.

	W_1	W_2	W_3	Supply
P_1	7	6	4	40
P_2	8	5	6	35
P_3	6	2	9	20
P_4	3	7	8	10
Demand	30	35	20	

(a) Find the Basic Feasible Solution for given transportation problem by using Least Cost Method, when supply from P_3 must be satisfied exactly.
(b) Check the optimality of this solution, if it is not optimal then find the optimal solution?
(c) Also find minimum cost. [12]

Q.2 Production manager needs to assign 4 jobs to 4 workers. The cost of performing a job is a function of the skills of the workers. Below table summarizes the cost (in Rs) of the assignments. Worker 1 cannot do job 3 and worker 3 cannot do job 4. Determine the optimal assignment using the Hungarian method.

	J1	J2	J3	J4
W1	50	50	-	20
W2	70	40	20	30
W3	90	30	50	-
W4	70	20	60	30

[10]

Q.3 An office equipment manufacturer produces two types of products: chairs and lamps. The production of either a chair or a lamp, requires one hour of production capacity in the plant. The plant has a maximum production capacity of 50 hours per week. Because of the limited sales capacity, the maximum number of chairs and lamps that can be sold are 6 and 8 per week, respectively. The gross margin from the sale of a chair is Rs 90 and from the sale of a lamp is Rs 60. The plant manager desires to determine the number of units of each product that should be produced per week in consideration of the following set of goals:

Goal 1: Available production capacity should be utilized as much as possible but should not exceed 50 hours per week.
Goal 2: Sales of two products should be as much as possible.
Goal 3: Overtime should not exceed 20 per cent of available production time.
Formulate this problem as a Goal Programming problem so that the plant manager may achieve his goals as closely as possible.

Note: Use notation s_i^+, s_i^- for **over-achievement** and **under-achievement** for the i^{th} goal.

[6]

Q.4 Solve the following LP problem by using M Method

$$\begin{aligned} \text{Maximize } z &= 5x_1 - 3x_2 + 4x_3 \\ \text{Subject to } & x_1 - 3x_2 - 2x_3 \leq 2 \\ & 5x_1 - 2x_2 + 4x_3 \geq 5 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$ [6]

Write the dual of the LPP above, find the solution of dual from primal optimal table. [6]

Q.5 Find the X and $f(X)$ after one iteration where $X = (x_1, x_2) \in \mathbb{R}^2$ for the given function, using the steepest ascent method

Maximize $f(X) = 4x_1 - 8x_1^2 + 12x_1x_2 - 10x_2^2$

take $(1, 1)^T$ as an initial guess. [10]

****END of PART A****

Birla Institute of Technology & Science, Pilani
First Semester 2023-2024, MATH F212 (Optimization)
Comprehensive Examination (PART-B Open Book)

Max. Marks: 40 **December 11, 2023** **Time: 90 Min.**

1. By branch and bound method, find an optimal solution and develop the branch and bound tree for the following integer programming problem:

$$\text{Maximize } z = 2x_1 + x_2$$
$$\text{subject to } x_1 + x_2 \le 5, \quad 6x_1 + 2x_2 \le 21,$$
$$x_1, x_2 \ge 0 \text{ and integer.}$$

[11]

2. Consider the problem: Maximize $f(x_1, x_2) = -2x_1^2 - x_2^2 + 8x_1 + 10x_2$ subject to

$$3x_1 + 2x_2 \le 6, \quad x_1 \ge 0, \; x_2 \ge 0.$$

(a) Either prove or disprove that f is concave. **[3]**

(b) What are KKT necessary conditions for this problem? **[5]**

(c) Are KKT necessary conditions obtained in (b) sufficient as well? Justify. **[1]**

(d) Find an optimal solution to this problem by quadratic programming algorithm. **[10]**

3. For the problem: Maximize $f(x_1, x_2) = x_1^3 + x_2^2 - x_1 + 2x_1x_2$, $0 \le x_1 \le 10$, $0 \le x_2 \le 10$, perform one iteration of unconstrained GA. Use three digits after decimal for your calculations. (The table should contain population strings, their decoded values, linearly mapped values, fitness function, A, B, C, D, E, F, mating pool, G, H, intermediate population, and mutation columns.) **[10]**

Use the following information to answer Question 3:

(a) The table containing the initial population and random numbers for selecting the mating pool is as follows:

Sr. No.	x_1	x_2	Random No.
1	10000	11110	0.851
2	10110	10011	0.544
3	11001	10110	0.021
4	00100	00100	0.786
5	10011	11100	0.815
6	01100	00011	0.540

(b) The probability of crossover is $p_c = 0.85$, and the probability of mutation is $p_m = 0.05$.

(c) The Random numbers for crossover are 0.21, 0.91, 0.51, and the random numbers for crossover sites are 02, 04, 10.

(d) The random numbers for mutation are as follows (Each row has 15 numbers):

0.01	0.24	0.25	0.65	0.07	0.09	0.89	0.20	0.58	0.24	0.21	0.98	0.87	0.25	0.11
0.02	0.55	0.23	0.34	0.99	0.25	0.14	0.28	0.88	0.77	0.45	0.36	0.25	0.18	0.58
0.69	0.25	0.47	0.87	0.08	0.28	0.47	0.87	0.99	0.06	0.14	0.58	0.74	0.01	0.45
0.08	0.09	0.52	0.64	0.66	0.47	0.58	0.24	0.59	0.66	0.06	0.05	0.07	0.09	0.02

END of PART B