# BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI <br> K K BIRLA GOA CAMPUS 

## FIRST SEMESTER 2019-2020, Mid-Term Examination (Closed Book)

Course no.: MATH F213
Day: Saturday
Date: September 28, 2019
Course title: Discrete Mathematics
Time: 90 minutes
Max Marks: 60
INSTRUCTIONS:1. All questions are compulsory. 2. Begin answering a new question on a fresh page. 3. Explain all steps clearly to get full credit. 4. Incomplete/No index costs 2 marks.

1. Translate the following into symbolic form and test the validity of the argument without using truth table.
If 6 is even then 2 does not divide 7. Either 5 in not prime or 2 divides 7 . But 5 is prime, therefore, 6 is odd.
2. Prove by principle of mathematical induction that for all integers $n \geq 4,3^{n}>n^{3}$.
3. Let $A=\{1,2, \cdots, 15\}$ and the equivalence relation $R$ on $A \times A$ defined by $(a, b) R(c, d)$ if $a d=b c$. Find the equivalence class of $(3,2)$.
4. Let $A=\{1,2,3,4\}$ and let $R$ be the relation on $A$, where $R=\{(1,2),(2,3),(3,4),(2,1)\}$. Find the transitive closure of $R$ using Warshall's algorithm.
5. Prove that for a bounded distributive lattice $L$, the complements are unique if they exist.
6. Consider the lattice $D_{60}=\{1,2,3,4,5,6,10,12,15,20,30,60\}$
(a) Find all join irreducible elements.
(b) Draw the Hasse diagram.
(c) Find all atoms.
(d) Find complements of 10 , if exist.
7. (a) Write the circuit diagram (or gate diagram) of $f\left(x_{1}, x_{2}, x_{3}\right)=\left(x_{1} \cdot x_{2}+x_{3}\right) \cdot\left(\left(x_{2}+x_{3}\right)+x_{3}\right)$. [4]
(b) Simplify the function in part- (a) by using basic Boolean algebra laws.
(c) Write the circuit (gate) diagram of the result obtained in part-(b).
8. (a) Use a K-map to find a minimal sum-of-products form for $E=x y^{\prime}+x y z+x^{\prime} y^{\prime} z^{\prime}+x^{\prime} y z t^{\prime}$.
(b) Find all prime implicants of $E=x y+y^{\prime} t+x^{\prime} y z^{\prime}+x y^{\prime} z t^{\prime}$ using consensus method.
