BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
First Semester 2022-2023
MATH F213 (Discrete Mathematics)
Comprehensive Examination (Part A : Closed Book)

Max. Marks : 50 **Max. Time : 105 mins (= 1hr 45 mins)**

- Notations/symbols have their usual meaning.
- Start new question on a fresh page. Moreover, answer each subpart of a question in continuation.

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1. (a) Find the general solution of the IHR
   $a_n - 5a_{n-1} + 6a_{n-2} = 3^n + 3^{n-1}4 + 3^{n-2}4^2 + \cdots + 3^{n-i}4^i + \cdots + 4^n; n \geq 2.$ [10]

   (b) Using an appropriate substitution, find the general solution of
   $a_n + 4na_{n-1} + 4n(n-1)a_{n-2} = 0; n \geq 2.$ [4]

2. Construct two digraphs which have same degree spectrum but they are not isomorphic. Justify your answer. [6]

3. Prove or disprove :

   (a) For a relation R on a nonempty set A, if $aRa\ \forall a \in A$ and for all $a, b, c \in A, aRb, bRc$ implies aRc, then R is symmetric. [4]

   (b) Let R be a symmetric and transitive relation on a nonempty set A. Show that if for all $a \in A$ there exists $b \in A$ such that aRb, then R is an equivalence relation on A. [4]

4. Let A be the set of all real valued functions on the real interval $[0, 2]$ and R be a relation on A defined by: for any $f, g \in A, fRg$ if and only if $f(x) \leq g(x)\ \forall x \in [0, 2]$.

   (a) Show that R is a partial order on A. [4]

   (b) Let $f, g \in A$ be given by $f(x) = x\ \forall x \in [0, 2]$ and $g(x) = x^2\ \forall x \in [0, 2]$. Find the join $f \vee g \in A$ if it exists, otherwise justify why it does not exist. [4]

5. Let $P(\{0, 1\})$ be the power set of $\{0, 1\}$ and let $A = \{0, 1\} \times P(\{0, 1\})$. Define a partial order R on A by $(x, S)R(y, T)$ if and only if $x < y$, or $x = y$ and $S \subset T$ for any $(x, S), (y, T) \in A$.

   (a) Draw the Hasse diagram of (A, R). [4]

   (b) Find the set of all upper bounds of $\{(0, \{0\}), (1, \emptyset)\}$. [4]

6. Let $A = \{a, b, c, d\}$ and $R = \{(a, b), (b, c), (c, c), (c, b)\}$ be a relation on A. Find R^k for all $k = 1, 2, 3, 4$ and hence the transitive closure of R. [6]

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BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
First Semester 2022-2023
MATH F213 (Discrete Mathematics)
Comprehensive Examination (Part B : Open Book)

Max. Marks : 40 **Max. Time : 75 mins (= 1hr 15 mins)**

- Notations/symbols have their usual meaning.
- Start new question on a fresh page. Moreover, answer each subpart of a question in continuation.

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1. Symbolize the following inference and check for its validity. Justify each step.

   A student in Section 1 of the course has not submitted the assignment.
   A student in Section 2 of the course has not submitted the assignment.
   Every student in Section 2 passed the mid-semester exam.

   Therefore, someone who passed the mid-semester exam has not submitted the assignment.

   [10]

2. A bookseller sells at least 1 book each day for 100 consecutive days, selling a total of 150 books. Prove that for every n with $1 \leq n \leq 49$, there is a period of consecutive days during which a total of n books were sold. [10]

3. Find the number of solutions of the equation

$$x_1 + x_2 + \cdots + x_{11} = -300, \ -50 \leq x_i \leq 100 \ \forall \ i = 1, \ldots, 11$$

   (no need to simplify binomial coefficients.) [10]

4. For $n \geq 0$, let $a_n = F_{3n}$, the $3n$-th Fibonacci number. Find (and prove) a second order linear recurrence relation with constant coefficients for (a_n). Hence find a closed form for the generating function for (a_n) using the method of generating functions. [10]