Birla Institute of Technology and Science, Pilani (Pilani Campus)
First Semester 2022-23
MATH F213 Discrete Mathematics
Mid-Semester Examination

Time: 90 mins **Date: 4-11-2022** **Max. Marks: 70**

Note. The total number of questions is 8. All questions are compulsory.

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1. Consider the proposition :

   Good batting as well as bowling is necessary for the team to be a winner or a runner.

   Let p : The team has a good batting, q : The team has a good bowling, r : The team is a winner, s : The team is a runner.
   Answer the following with justification.
   (A) Write symbolically a proposition equivalent to the opposite of above proposition using p, q, r, s and the connectives negation and disjunction only. **[3]**
   (B) Assume the converse of the original proposition has the truth value F. If possible then determine the truth value of the following proposition, otherwise explain why it is not possible :

   Good batting is sufficient for a team to be a winner, or good bowling is sufficient for the team to be a runner. **[5]**

2. Is the following inference valid? Justify your answer using the abbreviated truth table method.

   If A is a winner then B or C is a semifinalist. If B is a semifinalist then A is not a winner. If D is a semifinalist then C is not a semifinalist.
   Therefore if A is a winner then D is not a semifinalist **[8]**

3. There are 30 Mathematics, 20 Physics, 9 Chemistry, 15 Geography and 9 Philosophy books only in a library. Answer the following questions with justification :
   (A) What is the least number n such that randomly chosen n books of the library contain at least 10 books of same subject? **[5]**
   (B) What is the least number n such that randomly chosen n books of the library contain at least 10 books of Mathematics or at least 2 books of Physics? **[4]**

4. By mathematical induction on n, show that for any integer $n \geq 26$, there exist non-negative integers x, y such that $n = 4x + 7y$. **[10]**

5. Let $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$. For $n \geq 0$, solve for the entries of A^n using recurrence relations. Assume $A^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. **[10]**

6. Find a recurrence relation for a_n, the number of ways a sequence of 1's and 3's can sum to n. (Here, order matters: for example, $a_4 = 3$ as 4 can be obtained by the sequences $1111, 13, 31$.)

[4]

7. Find the general solution of the following recurrence relation by making an appropriate substitution to transform it into a linear recurrence relation with constant coefficients.

$$(n+1)^2 a_n - n^3 a_{n-1} = (n+1)!$$

[10]

8. Use generating functions to solve the recurrence relation

$$a_n - 2a_{n-1} - a_{n-2} + 2a_{n-3} = 0,$$
$$a_0 = 1,\ a_1 = 2,\ a_2 = 0.$$

[11]
