Time : 90 mins   Date : 10-10-2023   Max. Marks : 70

Instructions : 1) The question paper has **2 parts**, each of **35** marks and maximum **45** minutes. 2) Part B question paper and answer sheet will be distributed after the submission of Part A answer sheet. 3) Calculators are not allowed.

35 marks   <u>Part A</u>   **45 mins**

1. (a) Using the symbols p : f is continuous at a, q : $f(a)$ exists, r : $\lim_{x \to a} f(x)$ exists, s : $\lim_{x \to a} f(x) = f(a)$; write symbolically the statement
   $(*)$: For the continuity of a function f at a, it is sufficient that $f(a)$ exists, $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$.   [2]
   (b) Write a proposition equivalent to the negation of the opposite of $(*)$ using only p, q, r, s and connectives negation and conjunction. Justify all the steps.   [6]

2. (a) Using the method of abbreviated truth tables, determine if the following propositional function is a tautology or a contradiction or a contingency :
   $[(p \to q) \wedge (r \to s) \wedge (p \vee r)] \to (q \vee s)$.   [5]
   (b) If the following inference is valid, prove its validity using the rules of inference, otherwise prove its invalidity by giving the required combination of the truth values of the propostional variables :
   $(\sim t) \to (\sim r)$
   $\sim s$
   $t \to w$
   $r \vee s$
   ‒ ‒ ‒ ‒ ‒ ‒ ‒ ‒ ‒ ‒
   $\therefore w$.   [6]

3. The total count of apples in 10 boxes is 125. Using Pigeon hole principle, show that there will be at least one pair of boxes whose total number of apples is at least 25 and there is at least one pair of boxes whose total number of apples is at most 25.
   (Hint : If for $1 \leq i < j \leq 10$, $Y_{\{i,j\}}$ denotes the total noumber of apples in i^{th} and j^{th} box, first determine $\sum_{1 \leq i < j \leq 10} Y_{\{i,j\}}$.)   [8]

4. Write the following inference symbolically using the predicates $C(x)$: x is a member of the committee, $W(x)$: x is wealthy, $R(x)$: x is a republican, $O(x)$: x is old, and universe of discourse $U = x$ is an American :
   Every member of the committee is wealthy and a republican. Some committee members are old.
   Therefore there are some old republicans.

   If the inference is valid, establish its validity using the rules of inference, otherwise establish its invalidity.   [8]

Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2023-2024
MATH F213 Discrete Mathematics
Mid-semester Exam (Closed Book)

Time: 45 Minutes   Date: October 10, 2023   Max. Marks: 35

PART-B

Q.1 Using mathematical induction, prove that, for any integer $n \geq 1$, $2^{2n} - 1$ is divisible by 3.   [6]

Q.2 A sequence of decimal digits is a valid codeword if it contains an even number of 0 digits. Find a recurrence relation for computing the number of valid n-digit codewords. Hence, derive its solution using generating function.   [11]

Q.3 Determine the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$ (final answer should be in the form $\frac{a}{b^r} + \frac{c}{d^s}$ where a, b, c, d, r, s are integers).   [9]

Q.4 Using generating function, find the number of selections of 10 letters from "F, U, N, C, T, I, O" that contain at most 3 $U's$ and at least one O (final answer can be left in the form of $\binom{n}{r}$).   [9]