Birla Institute of Technology and Science, Pilani
First Semester 2023-24
MATH F213 (Discrete Mathematics)
Comprehensive Examination (Part A : Closed Book)
Time : 90 mins **Date : 9-12-2023** **Max. Marks : 45**

Instructions : 1) The question paper has **2 parts**, each of 45 marks and maximum 90 minutes.
2) Part B will be distributed after submission of Part A along with supplement for rough work. The question paper is built in, answers must be written in the space provided only.
3) Calculators are not allowed in the whole examination.

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1. Symbolize the following inference using predicates $P(x)$: x loves proofs, $D(x)$: x is in Discrete Mathematics class, $C(x)$: x has taken Calculus; and the universe of discourse $U =$ the set of all sudents in the Institute:

   Everyone in Discrete Mathematics class loves proofs.
   Someone in Discrete Mathematics class has not taken Calculus.
   Therefore someone who loves proofs has not taken Calculus.

   Also determine if the inference is valid or faulty. If valid, prove using rules of inference, otherwise justify using Venn diagrams. [8]

2. Prove, by mathematical induction, that for any integer $n \geq 2$,
   $\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+k} + \cdots + \frac{1}{2n} > \frac{13}{24}$. [7]

3. Find a recurrence relation along with initial conditions to determine the number of ways to make a pile of n chips using green, gold, red, white and blue chips such that no two gold chips are neighbours. (Here ways to make piles are distinguished by just looking at the orders in which colours of chips occur.) [7]

4. The generating function of the sequence $\{a_n\}_{n=0}^{\infty}$ is $f(x)$. Find the generating function $g(x)$ of $b_n = (-2)^n a_{n+1} + n; n \geq 0$ in terms of $f(x)$; where $a_0 = 0$. [7]

5. Find the general solution of $a_n - 2a_{n-1} + a_{n-2} = 5 + 3^n; n \geq 2$ by using the method of undetermined coefficients. [8]

6. Using appropriate substitution, find the general solution of the recurrence relation $a_n - 2a_{n-1}a_{n-2}^2 = 0; n \geq 2$. Assume $a_n > 0 \ \forall \ n \geq 0$. [8]

Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2023-2024

MATH F213 (Discrete Mathematics)

Comprehensive Examination (Part B: Open Book)

Time: 90 Minutes **Date: December 9, 2023** **Max. Marks: 45**

Note 1: Answer each question in the given space only with justification.

Name: **ID:**

Q.1 Determine if the inference is valid or faulty. If valid, then produce some evidence which will confirm the validity. If faulty, find a combination of truth values that will confirm a fallacy. **[5]**

If my plumbing plans do not meet the construction code, then I cannot build my house.

If I hire a licensed contractor, then my plumbing plans will meet the construction code.

I hire a licensed contractor.

$\therefore$ I can build my house.

Answer.

Q.2 Let $S = \{0, 1\}$. Consider the partial order R defined on $S \times S$ such that $(a, b)R(c, d) \iff$ either $a < c$ or both $a = c$ and $b \leq d$. Find the greatest, maximal, least and minimal elements of R. **[4]**

Answer.

Q.3 Let R be a partial order defined on set $\mathbb{Q}^+$ such that $aRb \iff \frac{a}{b} \in \mathbb{Z}^+$. Draw a Hasse Diagram representing the partial order R on the set $A = \{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 6\}$. Find all the complements of $\frac{1}{2}$ and 1 in the lattice $[A, R]$, if they exist. **[6]**

Answer.

Q.4 Let $S = \{\{\phi\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$. Prove or disprove: the lattice $[S, \subseteq]$ is distributive. **[5]**

Answer.

Q.5 Consider the Hasse diagram of a partial order R in Figure 1A. Write down indegree and outdegree of the vertices a, f in the directed graph of R. **[6]**

Answer.

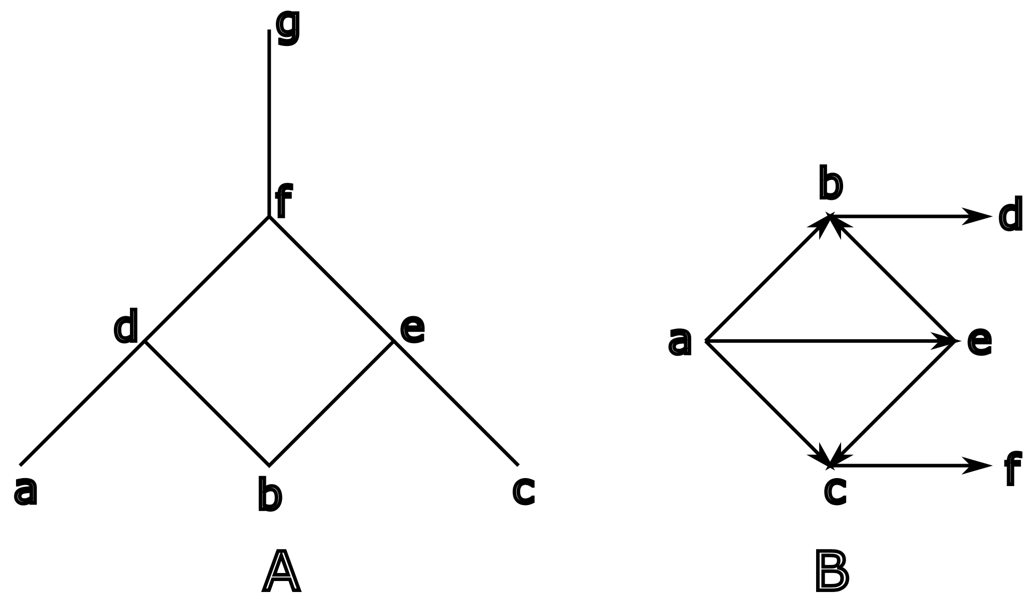

Figure 1:

Q.6 Write down all the unilaterally connected components of the digraph in Figure 1B. **[5]**

Answer.

Q.7 Let R be a reflexive relation on a non-empty set A. Prove or disprove: $R \subseteq R^2$. **[3]**

Answer.

Q.8 For the relation $R = \{(a, b), (a, c)\}$ find the transitive closure of the symmetric closure of R. **[5]**

Answer.

Q.9 Prove or disprove: The digraph of a partial order has no cycle of length greater than 1. **[6]**

Answer.