Birla Institute of Technology and Science, Pilani-K.K. Birla Goa Campus
First Semester 2019-2020
Midterm Examination (Closed Book)

Course No: MATH F214 Elementary Real Analysis Max. Mark: 60
Date: 05/10/2019 Time: 90 Min

Instructions:
(a) All questions are compulsory. (b) Start a new question on a fresh page. (c) Write all steps clearly and give proper explanation for complete credit. (d) Number all the pages of your answer book and make a question-page index on the front page.(e) **Incomplete or no index will cost 2 marks.**

1. Let A be the set of all real sequences whose elements are digits 1 and 2. Show that A is uncountable.

(10 marks)

2. Let (X, d) be a metric space and E be a subset of X. If p is a limit point of E, then show that every neighborhood of p contains infinitely many points of E.

(10 marks)

3. Show that compact subsets of metric spaces are closed.

(10 marks)

4. Let $a_n > 0$ and $\sum a_n$ be divergent. Show that
   (a) $\sum \left(\frac{a_n}{1 + n^2 a_n} \right)$ is convergent.
   (b) $\sum \left(\frac{a_n}{1 + a_n} \right)$ is divergent.

(4+6 marks)

5. If $a_1 = 3$ and $a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n}\right)$ for $n \in \mathbb{N}$. Show that $\{a_n\}$ is convergent and find its limit.

(10 marks)

6. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers. If $\sum a_n$ converges, and if $\{b_n\}$ is monotonic and bounded, prove that $\sum a_n b_n$ converges.

(10 marks)

THE END