**BITS PILANI, K K BIRLA GOA CAMPUS**
**Department of Mathematics**
**Mid-Test**
**MATH F214 Elementary Real Analysis**

Date: November 2, 2022 Marks: 35
Time: 2:00 - 3:30 PM.

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(1) (a) Define an *ordered field* $\mathbb{F}$ with an order relation $<$.
(b) Show that, if $z \in \mathbb{F}$ and $z < 0$, then $-z > 0$.
(c) Show that, if $x, y, z \in \mathbb{F}$ such that $x < y$ and $z < 0$, then $xz > yz$. (Marks: 2+2+1 = 5)

(2) Let $a \in \mathbb{R}$ with $a > 0$. Show that
(a) $a < 1$ implies $a^n \to 0$
(b) $a > 1$ implies $a^n \to \infty$. (Marks: 3+2 = 5)

(3) Let $E \subseteq \mathbb{R}$ be a bounded infinite set. Show that $E$ contains a convergent sequence with distinct entries. (Marks: 5)

(4) (a) Let $x_n \geq y_n \geq 0$ for all $n \in \mathbb{N}$. Prove that if $\sum_{n=1}^{\infty} x_n$ converges, then $\sum_{n=1}^{\infty} y_n$ converges. (You may use the fact: Every bounded monotonically increasing sequence of real numbers converges.)
(b) Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. (Marks: 2+3 = 5)

(5) Let $(X, d)$ be a metric space.
(a) Define: (i) Open set, (ii) Closure point of a set, (iii) Boundary point of a set.
(b) Show that every open ball in $X$ is an open set. (Marks: 3+2= 5)

(6) Let $(X, d)$ be a metric space and $A \subseteq X$. Show that $A$ is a closed set iff $A$ contains all its boundary points. (Marks: 5)

(7) Justify the following:
(a) The set $A = (0, 1]$ is neither open nor closed with respect to the usual metric on $\mathbb{R}$.
(b) Every subset of a discrete metric space is open and closed. (Marks: 3+2= 5)

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