BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
First Semester, 2023-24
Midsem Examination (Closed Book)
MATH F214 : Elementary Real Analysis

Max. Time: 90 Minutes | October 14, 2023 | Max. Marks: 60

1. State whether the following statements are true or false. Write the answers on the back of the front page of the answer sheet. $[1 \times 10 = 10]$
   (a) If $S \subseteq T$, then $\inf S \leq \inf T$.
   (b) If E is a nonempty, bounded subset of $\mathbb{Q}$, then $\inf(E) \in \mathbb{Q}$.
   (c) Every bounded sequence is monotone.
   (d) If (x_n) be a convergent sequence such that $x_n < \alpha$ for all n, then $\lim x_n \leq \alpha$.
   (e) Let $x_1 = 1$ and $x_{n+1} = 3x_n^2$, then $\lim x_n$ is either 0 or 1/3.
   (f) $\sum[\sin \frac{n\pi}{3}]^n$ is a convergent series.
   (g) The set $\{a + \sqrt{2}b : a, b \in \mathbb{Q}\}$ is countable.
   (h) Let E^o denote the set of all interior points of a set E. Every point of E^o is a limit point of E.
   (i) Let $\{G_\alpha\}$ be a collection of open sets, then $\bigcap\limits_{\alpha} G_\alpha$ is open.
   (j) Every finite subset of a metric space is compact.

2. (a) For $x_n = \frac{6n+4}{7n-3}$, find $\limsup x_n$ and $\liminf x_n$. [4]
   (b) Let (x_n) be a sequence such that $|x_{n+1} - x_n| < 2^{-n}$ for all $n \in \mathbb{N}$. Prove that (x_n) is a Cauchy-sequence. [8]

3. (a) Check the convergence of the series $\sum(\sqrt{n+1} - \sqrt{n})$. [6]
   (b) State and prove the alternating series test for the convergence of a series. [7]

4. Let A be the set of all sequences whose elements are the digits 0 and 1. Prove that A is uncountable. [6]

5. (a) Let (X, d) be a metric space and $E \subset X$. Define limit point, interior point and isolated point of the set E. [3]
   (b) Show that a set E is open if and only if its complement is closed. [8]
   (c) Prove that a closed subset of a compact set is compact. [8]

*** All The Best ***