Birla Institute of Technology and Science, Pilani
MATH F215; Algebra 1
First Semester 2017-2018
Mid Semester Examination [Closed Book]

Max. Marks: 60 **Max. Time: 90 Minutes** **Date: Oct.12, 2017**

Instructions:

- This question paper has **SIX** questions.
- Answer **ALL** questions.
- Answers to all subparts of a question should appear together.
- Support your conclusions always with proper explanations.

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1. Prove or disprove the following:

   (i) Let $\mathbb{Q} = \left\{\frac{m}{n}, m, n \in \mathbb{Z}, n \neq 0\right\}$. Then the group $(\mathbb{Q}, +)$ is cyclic. **[3]**

   (ii) There exist a group homomorphism ϕ from $\mathbb{Z}_{36}$ to $\mathbb{Z}_{20}$ such that $\phi([1]) = [2]$. **[3]**

   (iii) The alternating group A_4 has a subgroup of order 6. **[5]**

   (iv) If a group contains exactly one element of order 2, then that element is in the center of the group. **[5]**

2. For any group G, prove that the group $I(G)$ of inner automorphisms of G is isomorphic to the quotient group $\frac{G}{Z}$, where Z is the center of group G. Hence, determine the number of inner automorphisms of the symmetric group S_3. **[8 + 5]**

3. State and prove Cayley's theorem. **[10]**

4. Let G be a group and $f : G \to G$ defined by for all $a \in G$, $f(a) = a^3$ be an isomorphism. Prove that G is abelian. **[8]**

5. Show that a group of order 30 is not simple. **[7]**

6. Prove that every abelian group of order 6 is cyclic. **[6]**

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