BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
K K BIRLA GOA CAMPUS
COMPREHENSIVE EXAM, ALGEBRA-I, MATH F215
I SEMESTER – 2022-2023

Date: 17/12/2022 Max. Marks: 80 Duration: 180 mts

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1. Determine whether the permutation (1 2 3 4 5 6)(1 2 3)(4 5) is even or odd. [4]

2. Define the center $Z(G)$ of a group G. If $o(G) = p^n$ where p is a prime number then prove that $o(Z(G)) > 1$. [2+8]

3. Write all the conjugate classes of S_3. [6]

4. Prove that

   (a) Every field is an integral domain.

   (b) Every finite integral domain is a field. [4+6]

5. Prove that every homomorphic image of a commutative ring is isomorphic to a quotient ring. [10]

6. Let R be the commutative ring of all real valued continuous functions on the closed interval $[0, 1]$ under usual addition and product of functions. Let $M = \{f(x)|f(1/4) = 0\}$. Prove that M is an ideal of R. Is M a maximal ideal of R? Is M a prime ideal of R? justify your answers. [4+4+2]

7. Let R be a commutative ring with unity and I be an ideal of R. Prove that I is a prime ideal of R if and only if R/I is an integral domain. [6]

8. Define a Euclidean ring. Prove that the integral domain of Gaussian integers $\mathbb{Z}[i]$ is a Euclidean ring. [2+8]

9. State and prove Eisenstein criterion of irreducibility of polynomials. Using this, verify whether the polynomial $p(x) = 11x^4 + 15x^2 - 25x + 10$ is irreducible over the field of rational numbers or not. [2+5+3]

10. Is 13 a prime element in the Gussian ring of integers $\mathbb{Z}[i]$? Justify the answer. [4]

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