

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI  
K K BIRLA GOA CAMPUS  
COMPREHENSIVE EXAM, ALGEBRA-I, MATH F215  
I SEMESTER – 2022-23

Date: 31/10/2022

Max. Marks: 70

Duration: 90 min

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1. Prove or disprove that the following sets are groups under addition;
  - (a) the set of all rational numbers of absolute value  $\geq 1$  together with  $o$ .
  - (b) the set of rational numbers with denominators equal to 1 or 2. [10]
2. Write all the distinct right cosets of  $H = \{5m | m \in \mathbb{Z}\}$  in  $(\mathbb{Z}, +)$ . [6]
3. State and prove Lagrange's Theorem on groups. Prove or disprove the converse part of it. [12]
4. (a) If  $H$  and  $K$  are subgroups of a group  $G$  such that  $o(H) > \sqrt{o(G)}$ ,  $o(K) > \sqrt{o(G)}$  then prove or disprove that  $o(H \cap K) > 1$ .  
(b) If  $G$  is a group of order 77 then at most how many subgroups of order 11 does  $G$  may have? Give reasons. [10]
5. Define a normal subgroup of a group. Verify whether the subgroup  $\{(1), (1\ 3)\}$  of  $S_3$  is normal or not. [6]
6. Define a group homomorphism and prove that every homomorphic image of a group  $G$  is isomorphic to a quotient group of  $G$ . [12]
7. (a) Solve for  $x \in S_4$ , given  $(1\ 4\ 2)^2 \cdot x = (2\ 3\ 4)$ .  
(b) Find the order of
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 2 & 1 & 6 & 8 & 7 \end{pmatrix}.$$
[8]
8. (a) Verify whether  $(1\ 2)(3\ 4) \sim (2\ 3)(4\ 1)$  in  $S_4$ .  
(b) In  $S_7$  consider  $\sigma = (1\ 3\ 2\ 5\ 7)$  and  $\mu = (2\ 5\ 3\ 1\ 7)$ . Find  $\tau \in S_7$  such that  $\tau\sigma\tau^{-1} = \mu$ . [6]

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