Birla Institute of Technology & Science, Pilani
Second Semester, 2017-18
Make up Mid Semester Examination (Closed Book)

Course Name: MATH F231 (Number Theory) **Date: April 16, 2018 (Monday)**
Max. Time: 90 Minutes **Max. Marks: 35**

Note:1. Answer all sub-parts together.
2. Start new question from fresh page.
3. Symbols have their usual meaning.
4. Please write END at the end of the answer script.

Q1. State and prove the division algorithm theorem. **[6]**

Q2.(a) Determine all twin primes p and $q = p + 2$ such that $pq - 2$ is also a prime. **[3]**
(b) Prove that there are infinite numbers of primes of the form $6k + 5$ where k is positive integer. (Do not use Dirchlet's theorem.) **[4]**

Q3. Prove that the GCD of two positive integers can be written as a linear combination of them with integer coefficients. **[5]**

Q4. Let α, β be the roots of the quadratic equation $x^2 + mx - 1 = 0$, where m is an odd integer. Let $\lambda_n = \alpha^n + \beta^n$ for $n \geq 0$. Prove that λ_n is an integer and $(\lambda_n, \lambda_{n+1}) = 1$. **[5]**

Q5.(a) Prove that $(F_m, F_n) = F_{(m,n)}$ where $m, n \in \mathbb{N}$. **[4]**
(b) Derive Binet's formula. **[3]**

Q6. Prove that the linear diophantine equation (LDE) $ax + by = c$ is solvable iff $(a, b)|c$. Also prove that if x_0, y_0 is a particular solution of the above LDE then all solutions are given by $x = x_0 + \frac{b}{(a,b)}t$ and $y = y_0 - \frac{a}{(a,b)}t$, where t is an arbitrary integer. **[5]**

*******END*******