**Birla Institute of Technology & Science, Pilani**
**Second Semester, 2017-18**
**Mid Semester Examination (Closed Book)**

**Course Name: MATH F231 (Number Theory)** **Date: March 7, 2018 (Wednesday)**
**Max. Time: 90 Minutes** **Max. Marks: 35**

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**Note:1. Answer all sub-parts together.**
**2. Start new question from fresh page.**
**3. Symbols have their usual meaning.**
**4. Please write END at the end of the answer script.**

**Q1.** Prove or disprove:
(i) The sequence $71, 771, 7771, \ldots\ldots\ldots$ has a perfect square. **[3]**
(ii) For a given $a \in \mathbb{N}$ and $\forall\, n \in \mathbb{N}$, $a^2 | [(a+1)^{n+1} - an - (a+1)]$ **[2]**

**Q2.(a)** Let $p_n$ be the $n^{\text{th}}$ prime then prove that $p_{n+1} \leq p_1 p_2 \ldots\ldots\ldots p_n + 1$ and hence show that there are at least $n+1$ primes less than $2^{2^n}$ for $n \geq 1$. **[5]**
**(b)** Prove that there are infinitely many primes that do not belong to any pair of twin primes. **[2]**

**Q3.(a)** Let $a, b$ be positive integers and $d = (a, b)$ then $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$ **[3]**
**(b)** Prove that $(a, bc) = (a, (a, b)\, c)$, where $a, b, c \in \mathbb{N}$. **[3]**

**Q4.** Prove that any integer $\geq 2$, is either a prime or can be written as a product of prime numbers. Also prove that this factorization into primes is unique except for the order of the factors. **[5]**

**Q5.(a)** Prove that $F_{m+n} = F_{m-1}F_n + F_m F_{n+1}$ where $m \geq 2$ and $n \in \mathbb{N}$. **[4]**
**(b)** Using Binet's formula, prove that $F_{n+2}^2 - F_n^2 = F_{2n+2}$. **[3]**

**Q6.** A theatre charges ₹ 1.80 for adult admissions and ₹ 0.75 for children. On a particular evening the total receipts were ₹ 90. Assuming that more adults than children were present, find that how many people went to the theatre. **[5]**

*******END*******