Birla Institute of Technology and Science, Pilani (Pilani Campus)
Second Semester 2022-23

Course Number: MATH F231 (Number Theory)
Date of Examination: 15.03.2023 (Wednesday)
Maximum Duration: 90 min (9:00-10:30 am)

Examination: Mid-Semester Test
Mode: Closed Book
Maximum Marks: 70

Instructions.

1. Calculators are allowed. Symbols have their usual meanings.

2. If multiple answers are written for the same question, only the first one will be graded.

3. Answer each question legibly, clearly and concisely. Illegible answers will not be graded.

Q1. Determine if the following congruence in x, y, z is solvable. If so, find the complete solution.

$$x^3 + y^3 + z^3 \equiv 4 \pmod 9.$$ **[4]**

Q2. Find $\gcd(1947, 2023)$ and express it as a linear combination of 1947 and 2023.   **[8]**

Q3. Let $k, \ell \in \mathbb{N}$. Use induction to prove that $(k! \times \ell!)$ divides $(k + \ell)!$.   **[9]**

Q4. Let $m \in \mathbb{N}$ with $m \geq 2$. Prove that

$$\sum_{i=1}^{m} \frac{1}{2i - 1}$$

is not an integer.   **[9]**

Q5. Let $m, n \geq 2$. Prove that $\gcd(F_m, F_n) = F_{\gcd(m,n)}$.   **[10]**

Q6. Find the least positive integer x satisfying the following system of congruences.

$$x \equiv 13 \pmod{16},$$
$$x \equiv 9 \pmod{21},$$
$$x \equiv 19 \pmod{25}.$$

[10]

Q7. Let $n \in \mathbb{N}$. Prove that if
$$4((n - 1)! + 1) + n \equiv 0 \pmod{n(n + 2)},$$

then $(n, n + 2)$ is a twin prime pair.   **[10]**

Q8. Find all positive integers n such that $\varphi(n) = 2 \times 3^7$.   **[10]**

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ALL THE BEST

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