Birla Institute of Technology & Science, Pilani

Mid Semester Exam (Closed Book)

Course Name : Mathematical Methods (MATH F241) **Date: 07-03-18**

Max. Time: 90 Minutes **Max. Marks: 70**

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1. (i) Determine whether the function $g(x) = 1 - \dfrac{2\sin x}{[1-(\pi/2)]}$ is the solution or not of following integral equation: $g(x) - \int_0^{\pi} \cos(x+\xi) g(\xi) d\xi = 1$. [6]

   (ii) Obtain the boundary value problem corresponding to the following Fredholm integral equation

   $y(x) = \dfrac{5x}{6} + \dfrac{x^3}{6} + \lambda \int_0^1 K(x,\xi)\, y(\xi)\, d\xi$, where $K(x,\xi) = \begin{cases} \xi(1-x), & 0 \le \xi < x \\ x(1-\xi), & x < \xi \le 1 \end{cases}$. [6]

2. Prove that the Eigenvalues of real symmetric kernel $K(x,\xi)$ are real. [6]

3. Using Hilbert-Schmidt theory, solve the following integral equation :

   $y(x) = 1 + \lambda \int_0^{\pi} \cos(x+\xi)\, y(\xi)\, d\xi, \ \lambda \ne \pm \dfrac{2}{\pi}$, [10]

4. Using Neumann series (iterated kernal method) find the solution of following integral equation

   $y(x) = \dfrac{3e^x}{2} - \dfrac{xe^x}{2} - \dfrac{1}{2} + \dfrac{1}{2}\int_0^1 \xi\, y(\xi)\, d\xi$. [6]

5. Using the assumption $y(x) \approx c_1 + c_2 x + c_3 x^2$, find the approximate solution of $y(x) = x + \int_0^1 K(x,\xi)\, y(\xi)\, d\xi$, where $K(x,\xi) = \begin{cases} x(1-\xi), & x < \xi \\ \xi(1-x), & x > \xi \end{cases}$ using the method of weighting function (Galerkin Method) taking $1,\ x,\ x^2$ as weight function. [15]

6. State and prove the Fourier integral theorem. [10]

7. Find the inverse Fourier transform of $F(\xi) = e^{-a|\xi|}$, $a > 0$. [5]

8. Using Fourier sine transform, show that $\int_0^{\infty} \dfrac{x\sin x}{x^2+1} dx = \dfrac{\pi}{2e}$. [6]