Birla Institute of Technology & Science, Pilani

Comprehensive Exam **(Closed Book)** **Part A**

Course Name: Mathematical Methods (MATH F241) **Date: 05-05-18**

Max. Time: 90 Minutes **Max. Marks: 42**

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1. Prove that the Fourier cosine transform of $f = \cos(\frac{x^2}{2} - \frac{\pi}{8})$ is self-reciprocal. [8]

2. Find the 4-point inverse DFT of the discrete signal U_k with period 4 given by $U_k = \{0,0,4,0\}$. [4]

3. Derive the Euler's equation of the problem

$$I = \int_{x_1}^{x_2} F(x, y, y', y'')dx \quad \text{with} \quad \eta(x_1) = \eta(x_2) = \eta'(x_1) = \eta'(x_2) = 0.$$ [6]

4. Show that the solution of partial differential equation $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$ if $\frac{\partial U}{\partial x}(0,t) = 0$, $U(x,0) = \begin{cases} x, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$ and $U(x,t)$ is bounded where $x > 0, t > 0$ is $U(x,t) = \left(\frac{2}{\pi}\right)\int_0^\infty (\frac{\sin\xi}{\xi} + \frac{\cos\xi - 1}{\xi^2})e^{-\xi^2 t}\cos\xi x \, d\xi$. [8]

5. State and prove Hamilton's principle in its most general form. [8]

6. Find the extremal of isoperimetric problem $I[y(x)] = \int_0^\pi (y'^2 - y^2)dx$ subject to the constraint $\int_0^\pi y dx = 1$ under the conditions $y(0) = 0, y(\pi) = 1$. [8]