Birla Institute of Technology & Science, Pilani

First Semester 2021-2022, MATH F241 (Mathematical Methods)

Mid-Semester Examination

Time: 90 Min. Date: March 11, 2022 (Friday) Max. Marks: 70

1. Answer all parts of a question in continuation.
2. Write END in the answer sheet just after the final solution.

1. (a) Convert the IVP

$$y'' - \sin(x)y' + e^x y = x$$

with initial conditions $y(0) = 1, y'(0) = -1$ to a Volterra equation of the second kind. **[10]**

(b) Solve the equation $y'' = 1$ with given conditions $y(0) = y(1) = 0$ by using Green's function. **[10]**

(c) Determine the resolvent kernel for the Fredholm integral equation (IE)

$$y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 x\,t\,y(t)dt,$$

and hence, find the solution of the IE. **[12]**

2. (a) Consider the IE

$$\sin\frac{\pi x}{2} = \int_0^1 K(x,t)y(t)dt$$

where

$$K(x,t) = \begin{cases} x, & \text{when} \quad x < t \\ t, & \text{when} \quad x > t. \end{cases}$$

Obtain an approximate solution of the equation by assuming $y(x) \approx c_1 + c_2 x + c_3 x^2$, and using the method of collocation at the points $x = 0, \frac{1}{2}, 1$. **[12]**

(b) Apply the method of least squares to solve the integral equation

$$y(x) = 24 + \int_0^1 K(x,t)y(t)dt$$

where

$$K(x,t) = \begin{cases} x, & \text{when} \quad x < t \\ t, & \text{when} \quad x > t, \end{cases}$$

by assuming $y(x) \approx c_1 + c_2 x$. **[12]**

(c) Using Hilbert-Schmidt theorem, find the solution of the following symmetrical integral equation

$$y(x) = (x^2 + 1) + \lambda \int_{-1}^{1} (xt + x^2t^2)y(t)\,dt,$$

when λ is not an eigenvalue. **[14]**