Birla Institute of Technology & Science, Pilani

Mid Semester Exam (Closed Book)

Course Name : Mathematical Methods (MATH F241) Date: 13-03-23

Max. Time: 90 Minutes Max. Marks: 60

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1. Convert the following Volterra integral equation into the Initial Value Problem.

$$y(x) = 1 - x + \frac{x^3}{3} + \int_0^x [\sin\xi - (x-\xi)(e^{\xi} + \cos\xi)]y(\xi)\,d\xi \ . \qquad [6]$$

2. Using Fredholm theory, find the resolvent kernel associated with

$K(x,\xi) = 4x\xi - x^2, 0 \le x \le 1, 0 \le \xi \le 1.$

[6]

3. Using Hilbert-Schmidt theory, solve the following integral equation :

$$y(x) = 1 + \lambda \int_0^{\pi} \cos(x+\xi)\, y(\xi)\, d\xi,\ \lambda \ne \pm\frac{2}{\pi}, \qquad [11]$$

4. Using set of algebraic equations find approximate values of the solution of the equation $\frac{1}{2}(x^2 - x) + \int_0^1 K(x,\xi)\, y(\xi)\, d\xi = y(x)$, where $K(x,\xi) = \begin{cases} x(1-\xi), & x < \xi \\ \xi(1-x), & x > \xi \end{cases}$

taking $h=1/4$. Use the weighting coefficients of the Simpson's 1/3 rule. [14]

5. Using Fourier sine transform show that $\int_0^{\infty} \frac{x \sin x}{x^2+1} dx = \frac{\pi}{2e}$. [5]

6. Prove the Fourier integral theorem

$$\frac{1}{2}[f(x^+) + f(x^-)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x} \int_{-\infty}^{\infty} f(t) e^{i\xi t}\, dt\, d\xi$$ with usual notations. [8]

7. Find the inverse Fourier transform of $F(\xi) = \frac{e^{-i2\xi}}{(\xi-3)^2 + 5}$ [6]

8. Let $K(x,\xi)$ be real and symmetric kernel. Then, prove that the eigenfunctions corresponding to different eigenvalues are orthogonal. [4]