Birla Institute of Technology & Science, Pilani

Comprehensive Exam **(Closed Book)** **Part A**

Course Name : Mathematical Methods (MATH F241) **Date: 06-05-23**

Max. Time: 90 Minutes **Max. Marks: 46**

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1. Determine the resolvant kernel associated with $K(x,\xi) = x\xi$ in the interval (0,1) in the form of a power series in λ, obtaining the first three non zero terms. [6]

2. Find the Fourier transform of $e^{-9(x-4)^2}$. [6]

3. Given that $f[f(x)] = -2\sqrt{\frac{2}{\pi}}\left(\frac{\xi\cos\xi - \sin\xi}{\xi^3}\right)$, where $f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Using the inversion formula for Fourier transform, evaluate the integral $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\,dx$. [10]

4. Derive the Euler's equation of the problem

   $$I = \int_{x_1}^{x_2} F(x, y, y', y'')\,dx \quad \text{with} \quad \eta(x_1) = \eta(x_2) = \eta'(x_1) = \eta'(x_2) = 0.$$ [6]

5. Find the extremal of the functional $I[y(x)] = \int_{x0}^{x1} (y''^2 - 2y'^2 + y^2 - 2y\sin x)dx$. [6]

6. A particle of mass $2m$ falling vertically under the action of gravity and its motion is restricted by a force numerically equal to a constant c times of its square of distance x. Find the Lagrangian ***L*** and hence, find the equation of motion of the particle. [6]

7. Derive Hamilton's principle in its most general form under suitable assumptions. [6]

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Comprehensive Exam (Open Book) Part B

Course Name: Mathematical Methods (MATH F241) Date: 06-05-23

Max. Time: 90 Minutes Max. Marks: 44

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1. Solve the following integral equation:

$$y(x) = x + \lambda \int_{-\pi}^{\pi} (x\cos\xi + \xi^2 \sin x + \cos x \sin\xi)\, y(\xi)\, d\xi.$$ [6]

2. Using Rayleigh-Ritz method find the first approximate solution of the differential equation $(1-x^2)y'' - 2xy' + 2y = 0$ with $y(0) = 0, y(1) = 1$. (Use the assumption $y(x) \approx c_1 + c_2 x + c_3 x^2$ for the form of trail solution). [8]

3. Using calculus of variations, find the shortest distance between $x^2 + y^2 = 4$ and $2x + y = 6$. [10]

4. Find the extremal of isoperimetric problem $I[y(x)] = \int_0^1 (x^2 - y'^2)dx$ such that.

$I[y(x)] = \int_0^1 y^2 dx = 2$ and passing through the points $P(0,0)$ and $Q(1,0)$. [8]

5. By Fourier transform, evaluate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}, a > 0.$ [4]

6. Find the bounded solution of partial differential equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$ if $u(0,t) = 0, \quad u(x,0) = e^{-x}, x > 0$ and $U(x,t)$ is bounded where $x > 0, t > 0$.

[8]