Birla Institute of Technology & Science, Pilani

MATH F243: Graphs and Networks

End-Semester Examination (Closed Book)

Max. Marks: 50 **May 07, 2022** **Time: 105 Minutes**

Note: Calculators are not allowed. No marks will be awarded if proper justification is missing.

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1. Draw a maximal planar graph in which every vertex has degree exactly 5. **[8]**

2. Let G be a planar graph of order n such that every vertex of G has degree at least five, and at least one vertex of G has degree eight. Find the least k such that $n \geq k$ (Hint: try to use Euler Formula). **[12]**

3. 7 cricket teams, $A-G$, are required to play 13 matches as shown in Figure 1 (a cross indicates that those teams must play each other). The matches are to be scheduled so that no team plays more than one match in any week. Using the concepts of graph theory, compute the minimum number of weeks required. **[10]**

A	×	×	−	−	−	×
	B	×	×	−	×	−
		C	×	×	−	−
			D	×	×	−
				E	×	×
					F	×
						G

Figure 1:

4. Compute $\alpha'(G)$ and $\beta(G)$ for the graph G in Figure 2 (justify your answer). **[10]**

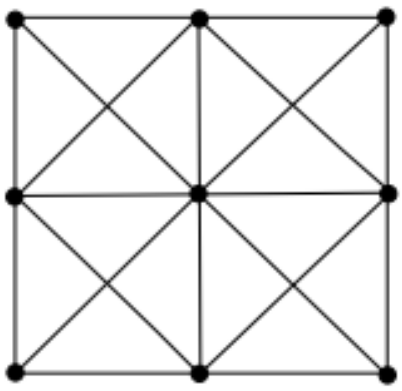

Figure 2:

5. Prove that a graph G is Hamiltonian if and only if its closure is Hamiltonian. **[10]**

End

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MATH F243: Graphs and Networks

End-Semester Examination (Open Book)

Max. Marks: 40 **May 07, 2022** **Time: 75 Minutes**

Note: Calculators are not allowed. No marks will be awarded if proper justification is missing.

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1. Using Menger's Theorem, prove that for every $k \geq 3$, $K_{k,k-2}$ is $(k-2)$-edge connected. **[10]**

2. Using Ford Fulkerson Algorithm, find a maximum flow from s to t for the network flow in Figure 1. Hence, find a minimum cut. **[8]**

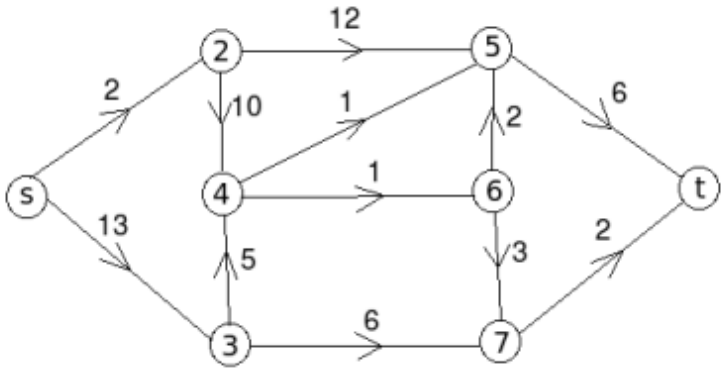

Figure 1:

3. Using Dijkstra's Shortest Path Algorithm, find the length s of the shortest path from A to I. For which edges e, will making e longer by 0.1 change s. **[12]**

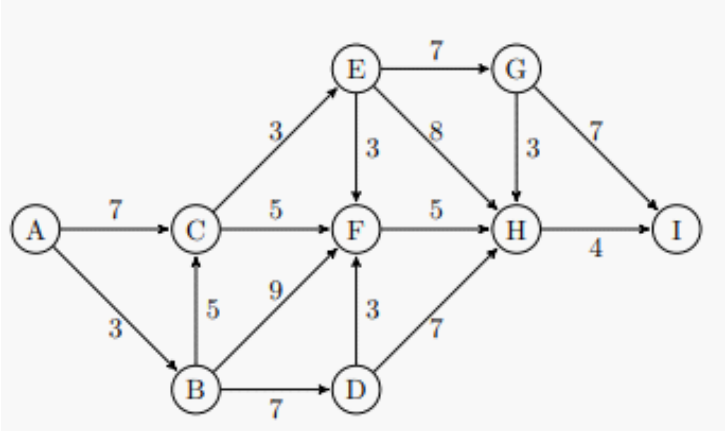

Figure 2:

4. Using induction, prove that every simple graph with $n \geq 7$ vertices and at least $5n-14$ edges contains a subgraph with minimum degree at least 6. **[10]**

End