Birla Institute of Technology & Science, Pilani (Raj.)

Second Semester 2021-2022, MATH F243

Mid-Semester Exam (Closed Book)

Time: 90 Minutes **Date: March 9, 2022** **Max. Marks: 70**

Q.1 Determine the number of spanning trees for the graph in Figure 1. How many of these are non-isomorphic as unlabelled spanning trees? **[12]**

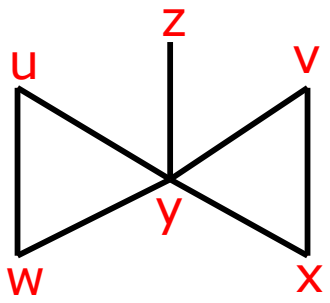

Figure 1:

Q.2 Use Kruskal's algorithm to construct a minimal spanning tree of the graph in Figure 2 (show all the steps). **[8]**

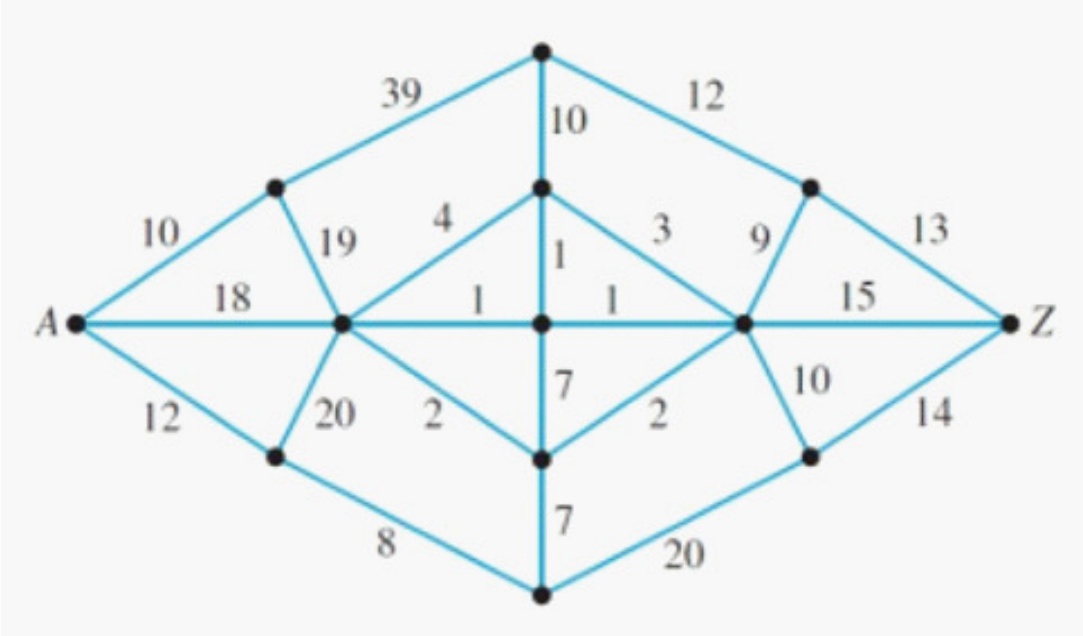

Figure 2:

Q.3 For the chess board in Figure 3 with 2 missing squares, consider a graph by taking each square to be a vertex and two squares are adjacent if they share a common edge. Is the obtained graph Hamiltonian (justify your answer). **[10]**

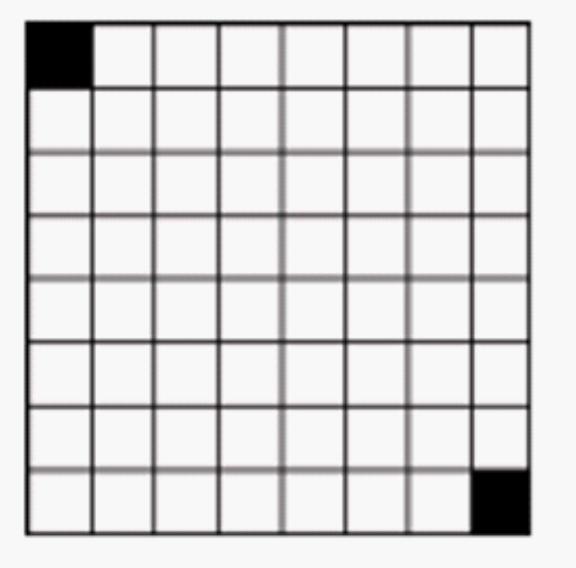

Figure 3:

Q.4 Draw all simple digraphs of order 4 and size 3 (it has no partial marking). **[10]**

Q.5 Prove by induction that the complete binary tree (binary tree with each vertex having exactly two children) of height h has $2^{h+1} - 1$ vertices. **[8]**

Q.6 Let G be an r-regular graph of odd order n. Prove or disprove: G^C is Eulerian assuming it is connected. **[10]**

Q.7 Prove that a k-regular simple connected graph G with girth 5 must have at least k^2+1 vertices (girth of a graph is the length of a shortest cycle contained in the graph). **[12]**