Birla Institute of Technology & Science, Pilani (Raj.)

Second Semester 2022-2023, MATH F243
Mid-Semester Exam (Closed Book)

Time: 90 Minutes   **Date: March 16, 2023**   **Max. Marks: 70**

Q.1 Suppose the average degree of the vertices of a simple connected graph G is exactly 2. How many cycles does G have? Support your answer. **[4]**

Q.2 List all simple distinct graphs with degree sequence $\langle 2, 2, 2, 2, 2, 2, 2, 1, 1 \rangle$. **[7]**

Q.3 Let T be a tree in which all vertices adjacent to leaves have degree at least 3. Prove or disprove that there exists a pair (u, v) of leaves that have a common neighbor. **[6]**

Q.4 Draw all non-isomorphic rooted trees with 4 edges (it has no partial marking). **[7]**

Q.5 For the graph in Figure 1, find the number of different walks of length 4 from v_5 to v_5. **[6]**
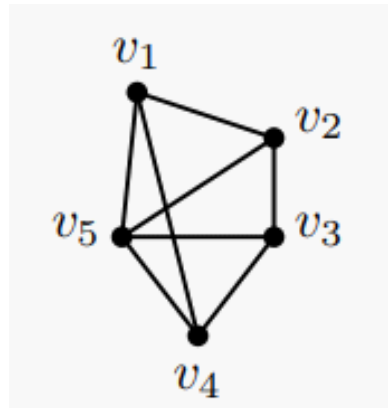

Figure 1:

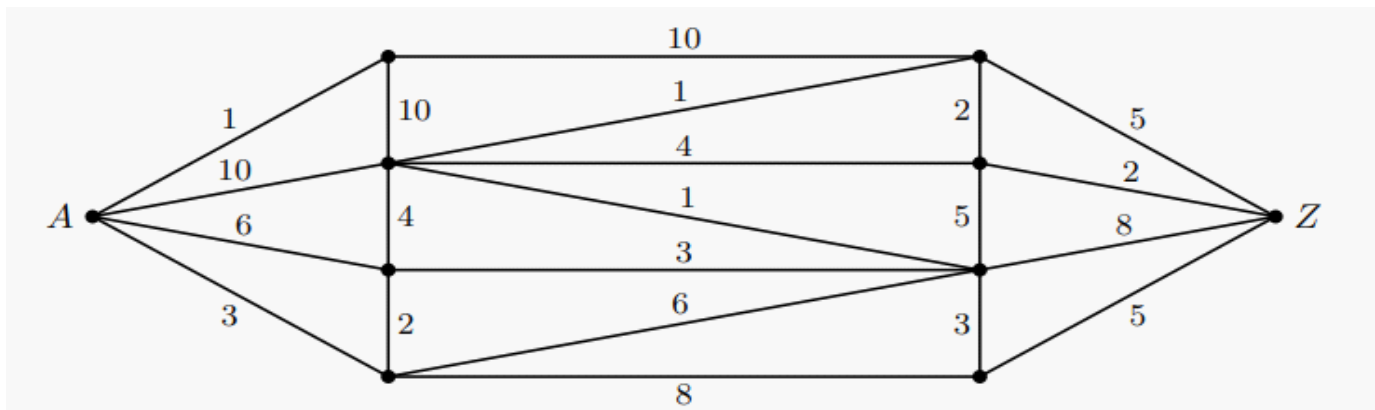**Q.6** Using Dijkstra's Shortest Path Algorithm, for the graph in Figure 2, find all shortest paths from A to Z. **[9]**

Figure 2:

Q.6 Let G be an k-regular graph of odd order n. Prove or disprove: the complement of G is Eulerian provided it is connected. **[8]**

Q.7 A club has seven members and meets for lunch each month. If the members sit at a round table and decide to sit so that each member has different neighbours at each lunch, determine how many months this arrangement could last, and give possible seating arrangements for these months. **[9]**

Q.8 Consider the digraph in Figure 3. With justification, answer the following:

    (a) Is it simple? **[2]**

    (b) Is it Eulerian? **[2]**

    (c) Is it Hamiltonian? **[2]**

    (d) Is it strongly connected? **[2]**

Figure 3:

Q.9 Draw a 3-regular simple connected graph G of order 10 such that $\kappa(G) = \kappa'(G) = 1$. **[6]**