Birla Institute of Technology & Science, Pilani

MATH F243: Graphs and Networks

End-Semester Examination (Open Book)

Max. Marks: 48 **May 15, 2023** **Time: 105 Minutes**

Note: Calculators are not allowed. No marks will be awarded if proper justification is missing.

Q.1 Let D be a simple digraph of order n, $n \geq 3$. Write the maximum and minimum number of arcs D can have (in terms of n), when D is weakly connected and strongly connected. **[8]**

Q.2 Compute the number of labelled trees on 5 vertices. **[8]**

Q.3 Compute the number of spanning trees (in terms of s) for $K_{2,s}$, $s \geq 2$. **[6]**

Q.4 Perform a depth first search on the tree in Figure 1, starting with vertex a (when there is a choice of vertices to visit, always visit the one which comes first in alphabetical order). **[4]**

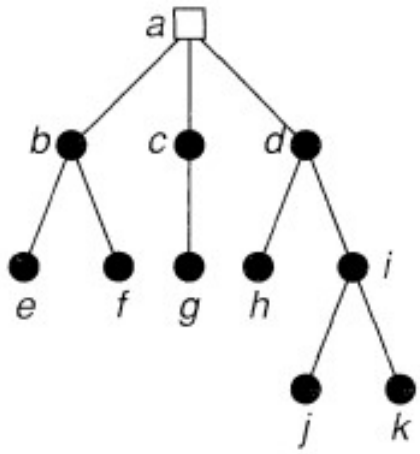

Figure 1:

Q.5 Let G be a simple connected graph with at least two vertices. Prove or disprove that $\kappa(G) \leq \frac{2m}{n}$. **[6]**

Q.6 Let A be the adjacency matrix of some graph G. Find $[A^k]_{i,j}$ for $1 \leq k < d(v_i, v_j)$. **[4]**

Q.7 For n odd, identify a class of graphs to show that the condition $\deg(v) \geq n/2$ in the statement of Dirac's theorem, cannot be replaced by $\deg(v) \geq (n-1)/2$. **[6]**

Q.8 Show how the analysis of the flows in a network with several sources and sinks can be reduced to the standard case by the addition of a new 'source vertex' and 'sink vertex'. **[6]**

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MATH F243: Graphs and Networks

End-Semester Examination (Closed Book)

Max. Marks: 42 **May 15, 2023** **Time: 75 Minutes**

Note: Calculators are not allowed. No marks will be awarded if proper justification is missing.

Q.1 A graph is called outerplanar if it has a drawing in which every vertex lies on the boundary of the outer face. Show that if a graph is outerplanar, then it contains neither K_4 nor $K_{2,3}$ as a minor. **[6]**

Q.2 Let G be a simple connected graph with n vertices and $n+2$ edges. Prove or disprove: G is planar. **[6]**

Q.3 Find the crossing number of $K_{4,3}$. **[6]**

Q.4 For a simple connected graph G with n vertices, let $\chi(G) = n$. By contradiction, prove that $G = K_n$. **[6]**

Q.5 Let G be a simple connected 3-regular Hamiltonian graph, then compute $\chi'(G)$. **[6]**

Q.6 Draw a simple connected, 3-regular graph that has both a cut vertex and a perfect matching. Also, highlight the perfect matching. **[6]**

Q.7 Let T be a tree of order 20 and 12 be the maximum size of an independent set in T. Compute $\alpha'(T)$. **[6]**

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