Birla Institute of Technology and Science, Pilani

II Semester 2021-2022

MATH F244: Measure & Integration

Mid Semester Exam (Closed Book)

Time: 90 Min. **Max. Marks: 60**

Q.1. Let $\boldsymbol{A}$ be the set of points in [0 1] such that x is in $\boldsymbol{A}$ iff the decimal expansion of x does not require the use of the digit 5 and 8. Determine **m*(*A*).** [10]

Q.2. Prove or disprove that if E is a measurable subset of $\mathbb{R}$, and $\bar{E}$ is closure of E

(a) E is countable then $m(E) = m(\bar{E})$.

(b) E is uncountable then then $m(E) = m(\bar{E})$. [5+5]

Q.3. Using the fact that $[a, \infty)$ is a measurable set, show that $[a, b]$ is measurable set. Write the statements of the theorems you are using. [10]

Q.4. Let $f: [0,1] \to \mathbb{R}$. Prove or disprove that if the set $\{x \in [0,1]: f(x) = c\}$ is a measurable subset of $\mathbb{R}, \forall c \in \mathbb{R}$ then f is a measurable function. [10]

Q.5. Show that a continuous function defined on a measurable set is a measurable function. Does the converse of this statement true? If "Yes" then prove and if "No" then give a counter example with full justification. [10]

Q.6. Define *simple function* and show that a *simple function* is *a measurable function.* [3+7]