Birla Institute of Technology and Science, Pilani
I-Semester 2016-17
(Introduction to Topology) MATH F311
Comprehensive Exams (Close Book)

Max. Marks 50 Date: 9th December 2016 Time: 100 Min.

Q.1. Define *Path connectedness* and show that "*Topologist sine curve*" is not path connected. [10]

Q.2. Define *quotient space* X^* for a topological space X. Show that if $f : X \to Y$ is a quotient map, then the relation $x \sim y$ if and only if $f(x) = f(y)$ is an equivalence relation on X and hence show that $Y \cong X^*$.
[10]

Q.3. Critically analyze the following statement:
"*A topological space is compact if and only if it is limit point compact*". [10]

Q.4. Define *first countable space*. Let X is *first countable space* and $A \subset X$. Show that if $x \in \bar{A}$, then there is a sequence of points of A converging to x. [10]

Q.5. Define *regular space*. Show that the space $\mathbb{R}_K$ is *Hausdorff*, but not *regular*.
[10]

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2015-2016

MATH F311 (Introduction to Topology)

Comprehensive Exams (Quiz)-Open Book

Max Marks: 30 **Date: 9th December 2016** **Time: 80 Min.**

ID…………………………………………... **Name………………………………**

Q.1. Consider *Sorgenfrey plane* and if $A=\{(x,y): x^2+y^2 \le 1\}$ then find *int* (A).

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Q.2. Is any *compact* T_2 space *normal*? Justify.

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Q.3. Let X be a *compact Hausdorff* space and let $f : X \to X$ is *continuous*. Define $X_1 = X$ and $X_{n+1} = f(X_n)$ and set $A = \bigcap_{k \in N} X_k$. Is A non-empty? Justify.

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Q.4. Let $\mathbb{N}$ be the set of all *natural numbers* and $\mathbb{B}$ be the collection of all *arithmetic progressions* on $\mathbb{N}$. Then $\mathbb{B}$ is a basis on $\mathbb{N}$. Let p be a *prime number*. Show that the set $F_p = \{np: n \in \mathbb{N}\}$ is *closed*.

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Q.5. If every *continuous* function $f : X \to \{0, 1\}$ is *constant*, then show that X is *connected*.

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Q.6. Let $\tau = \{G \subset \mathbb{R}: 0 \notin G\} \cup \{\mathbb{R}\}$ be a topology on $\mathbb{R}$ and $Y = \mathbb{R} \sim \{0\}$. Is (Y, τ_Y) *Lindelöf*? Justify.

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FIRST SEMESTER 2015-2016
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