BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2017-2018
MATH F311 (Introduction to Topology)
Comprehensive Exams (Part-A)-Close Book

Max Marks: 30 **Date: 13th December 2017** **Time: 60 Min.**

ID…………………………………………...	**Name……………………………………...**

Q.1. Fill in the blanks:

(i) Consider the basis $\mathbb{B} = \{[a, b) : a < b$ are real numbers$\}$ on $\mathbb{R}$, the topology induced by this basis is called ………………………………………topology, and $\mathbb{R}$ with this topology is called …………………………………… line.

(ii) Let $D = \{x \times y : x$ is rational$\} \subseteq \mathbb{R}^2$ with standard topology. Then $Int\,(D) =$………………………

and $\overline{D} =$ ………………………………

(iii) Ordered square (I_0^2) and $\mathbb{R}$ with the topology induced by $\mathbb{B}$ given in (i), both satisfy ……………………………………**{first countable/second countable/separable}** axiom, but not ……………………………………**{metrizable/normal}**

(iv) Let X be a topological space and let $A \subseteq X$, *exterior* of A is defined as $Ext\,(A) = X \sim \overline{A}$, then

$Ext\,(A \cup B)$ ………….. $Ext\,(A) \cap Ext\,(B)$ **{=/⊇/⊆}**

and $Ext\,(A) \cap A =$ …………………………

(v) Let $f : X \rightarrow Y$ and $A \subseteq X$, then f is an open mapping if and only if $f\,(Int\,(A))$ .... $Int\,(f\,(A))$ **{=/⊇/⊆}**

and f is a closed map if and only if $\overline{f\,(A)}$............$f\,(\overline{A})$ **{=/⊇/⊆}** **[2+2+2+2+2]**

Q.2. Let $< x_n >$ be sequence of points in a Hausdorff space X converges to a point $x \in X$. Let $y \neq x$, then show that $< x_n >$ does not converges to y. **[4]**

Ans:……………………………………………………………………………………………….
………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………
………………………………………………………………………………………………………

Q.3. Let $A \subseteq \mathbb{R}^{\omega}$ with box topology, where $A = \{(x_1, x_2, \ldots) : x_i > 0, \forall\, i \in \mathbb{N}\}$, then $\mathbf{0} \in \overline{A}$. Show that there is no sequence of points of A converging to $\mathbf{0}$. **[4]**

Ans:……………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………

Q.4. Prove or disprove that if a topological space is metrizable, then any coarser topology is also metrizable. **[4]**

Ans:……………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………

Q.5. Prove or disprove that if for all $x, y \in X$ there exists a connected set $C_{x,y} \subseteq X$, with $x, y \in C_{x,y}$, Then X is connected. **[4]**

Ans:……………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………

Q.6. Prove or disprove that $[0, 1]$ is compact as a subspace of $\mathbb{R}_K$. **[4]**

Ans:……………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………
……………………………………………………………………………………………………

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2017-2018
MATH F311 (Introduction to Topology)
Comprehensive Exams (Part-B)-Open Book

Max Marks: 50 **Date: 13th December 2017** **Time: 120 Min.**

Q.1. Show that $\mathbb{B} = \{[a, b], a < b, a, b \in \mathbb{R}\}$ will not form a basis for any topology on $\mathbb{R}$. **[5]**

Q.2. Let $(Y, \mathscr{T})$ be a non-normal topological space, let $p \notin Y$, let $X = Y \cup \{p\}$, and $\mathscr{T}_X = \mathscr{T} \cup \{X\}$. Show that $\mathscr{T}_X$ is a topological space on X. Is $\mathscr{T}_X$ normal? Justify. **[4+1+5]**

Q.3. Consider $X = \mathbb{Z}^{\omega}$, (countable product of $\mathbb{Z}$ with itself), where each $\mathbb{Z}$ has subspace topology of standard topology on $\mathbb{R}$.

(a) Is X with the box topology metrizable? Can the Uryshon metrization theorem be applied to conclude that X with the box topology is metrizable? State the necessary properties and determine whether or not they hold. **[1+1+4]**

(b) Is X with the product topology metrizable? Can the Uryshon metrization theorem be applied to conclude that X with the product topology is metrizable? State the necessary properties and determine whether or not they hold. **[1+1+4]**

Q.4. Let X be an uncountable set and let $p \in X$, consider the topology $\mathscr{T} = \{X\} \cup \{G \subseteq X : p \notin G\}$ on X. Show that $(X, \mathscr{T})$ is not separable. **[8]**

Q.5. Consider $\mathbb{R}$ with co-finite topology, prove or disprove that $\mathbb{Q} \cup \mathbb{P}$ is connected subset, where

$\mathbb{P} = \{\alpha : \alpha$ is a real root of $a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0 = 0$, $n \in \mathbb{N}$ and $a_i \in \mathbb{Z}, \forall\, i = 0, \ldots, n\}$. **[1+6]**

Q.6. Let X be an infinite set, let $a, b \in X$, consider the topology $\mathscr{T} = \{X\} \cup \{U \subseteq X : a, b \notin U\}$ on X. Let $A = X \sim \{a\}$ and $B = X \sim \{b\}$, show that A and B are compact subset of X. Is $A \cap B$ compact? Justify.

[4+1+3]