Birla Institute of Technology and Science, Pilani
I-Semester 2017-18
(Introduction to Topology) MATH F311
Mid-Semester Exams (Close Book)

Max. Marks 60 Date: 14th October 2017 Time: 90 Min.

Q.1 For each $n \in \mathbb{Z}$, define $B_n = \begin{cases} \{n\} & \text{, if } n \text{ is odd} \\ \{n-1, n, n+1\} & \text{, if } n \text{ is even} \end{cases}$. Show that $\mathbb{B} = \{B_n : n \in \mathbb{Z}\}$ is a basis for a topology on $\mathbb{Z}$. [8]

Q.2 Define "*Lexicographic order*" relation on $\mathbb{R} \times \mathbb{R}$. Show that the topology τ generated by this ordering on $\mathbb{R} \times \mathbb{R}$ is same as product topology $\mathbb{R}_d \times \mathbb{R}$. Where $\mathbb{R}_d$ is the set of real numbers with discrete topology and $\mathbb{R}$ is the set of real numbers with usual topology. Compare τ with the standard topology on $\mathbb{R} \times \mathbb{R}$. [14]

Q.3 Define *Hausdorff* space. Show that every *finite set* is *closed* in a *Hausdorff* space. [8]

Q.4 Let $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}\}$ be a topology on $X = \{a, b, c, d\}$, let $f : X \to X$ be a map defined as: $f(a) = b = f(c)$, $f(b) = d$, $f(d) = c$. Check the *continuity* of f at each point of X. [8]

Q.5 Show that the product space $\mathbb{R}^{\omega}$ with product topology is *metrizable* with the *metric* D, defined as: $D(x, y) = \sup\left\{\frac{\bar{d}(x_i, y_i)}{i}\right\}$, where $\bar{d}(x, y)$ is *standard bounded metric* on $\mathbb{R}$. [10]

Q.6 Define *quotient space* $X/\sim$ for a topological space X. Show that if $f : X \to Y$ is a *quotient* map, then the relation $x \sim y$ if and only if $f(x) = f(y)$ is an *equivalence* relation on X and hence show that $Y \cong X/\sim$. [12]

All the very best