BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2022-2023

MATH F311 (Introduction to Topology)

Mid Semester Exam (Closed Book)

Max Marks: 50 **Date: 4th November 2022** **Time: 90 Min.**

Q.1. Define *closure* of a set *A*. Prove or disprove following:

(*a*) If *A* and *B* are open subset of a topological space *X* s.t. $A \cap B = \phi$ then $\overline{A} \cap \overline{B} = \phi$

(*b*) Let *A* be subset of a topological space *X*, then $A \subset \overline{(\text{int } A)}$. [3+4+4]

Q.2. Define *continuity* of a function *f*. Let *X* and *Y* be sets, each with the *co-finite topology*. Prove that a function $f : X \to Y$ is *continuous* if and only if either *f* is a constant function or else $f^{-1}(\{y\})$ is finite for all $y \in Y$. [3+5]

Q.3. Write the statement (No need to prove) of "*The pasting lemma*". Prove or disprove that the function $f{:}\mathbb{R}_l \to \mathbb{R}_l$ defined as $f(x) = \left(\frac{x^3}{2} - x\right)$ is *continuous*. [3+1+5]

Q.4. Write the statement (No need to prove) of "*The sequence lemma*". Let $X = \mathbb{R}^\omega$ with the *box* topology. Let $A \subset \mathbb{R}^\omega$ consist of the points $(x_1, x_2, \ldots)$ with all $x_i > 0$.

(*a*) Show that $\mathbf{0} = (0, 0, \ldots)$ belongs to the closure of *A*.

(*b*) Prove or disprove that a sequence of points in *A* converges to **0**.

(*c*) Is $\mathbb{R}^\omega$ with the box topology metrizable? Justify your answer. [3+3+3+3]

Q.5. Define *standard bounded metric* $\overline{d}$ on a metric space (X, d). Prove or disprove that the topologies induced by *d* and $\overline{d}$ on *X* are same. [3+1+6]

*** *All the Best****