SET-X

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2022-2023

MATH F311 (Introduction to Topology)

Comprehensive Exam-Part A (Quiz) (Closed Book)

Max Marks: 30 **Date: 28th December 2022** **Time: 60 Min.**

ID………………………………………… Name…………………………………..................

1. **Write the most appropriate option from the following MCQ in the boxes below only, answer written anywhere else will not be checked. Any cross cutting overwriting will be considered as question not attempted.**

2. **Each question carries 3 Marks.**

Q. No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans.										

Q.1 If each space X_α is a Hausdorff space, then $\prod_{\alpha \in \wedge} X_\alpha$ is a Hausdorff space in:

[A] both *product* and *box* topologies
[B] *box* but not in *product* topology
[C] *product* but not in *box* topology
[D] neither in *box* nor in *product* topology

Q.2 Let $\tau = \{\Phi, X, \{a\},\{b\},\{a, b\},\{b, c, d\}\}$ be a topology on $X = \{a, b, c, d\}$. Let $f : X \to X$ be a map defined as: $f(a) = b = f(c), f(b) = d, f(d) = c$. Then f is continuous at:

[A] only d
[B] both a and b
[C] a but not at c
[D] only c.

Q.3. Let (X, d) be a metric space, the distance $\delta(x, A)$ between a point $x \in X$ and a subset $A \subset X$ is defined as $\delta(x, A) = \inf\{d(x, a): a \in A\}$, then A is closed in X if and only if:

[A] $\{x \in X: \delta(x, A) = 0\} \subset A$
[B] $\{x \in X: \delta(x, A) > 0\} \subset A$
[C] $\{x \in X: \delta(x, A) = 0\} \supset A$
[D] $\{x \in X: \delta(x, A) > 0\} = A$

Q.4. Let $\mathscr{T}_c$ be the co-finite topology on infinite set X:

[A] $(X, \mathscr{T}_c)$ is indiscrete space
[B] $(X, \mathscr{T}_c)$ is T_2-space
[C] $(X, \mathscr{T}_c)$ is discrete space
[D] $(X, \mathscr{T}_c)$ is T_1-space

Q.5. Which of the following is falls:

[A] Every metric space is compact Hausdorff space.
[B] Every metric space is normal space.
[C] Every metric space is regular space.
[D] Every metric space is Frechet space.

Q.6. For $A \subseteq \mathbb{R}^2$, let $\mathscr{T}$ be the subspace topology on $X = \mathbb{R}^2 \sim A$ then:

[A] A is countable $\Rightarrow (X, \mathscr{T})$ is connected.
[B] $(X, \mathscr{T})$ is connected $\Rightarrow A$ is finite.
[C] A is unbounded $\Rightarrow (X, \mathscr{T})$ is compact.
[D] A is open $\Rightarrow (X, \mathscr{T})$ is compact.

Q.7. The subspace $\{0, 1\}$ in $\mathbb{R}$ with usual metric is:

[A] connected **[B]** open **[C]** compact **[D]** clopen

Q.8. A subset of $\mathbb{R}$ which is both compact and connected is:

[A] A finite set **[B]** An open interval **[C]** A closed interval **[D]** A bounded interval

Q.9. Let $X = \{a, b, c\}$, let $\mathscr{T}_1 = \{\Phi, X, \{a\}, \{b, c\}\}$ and $\mathscr{T}_2 = \{\Phi, X, \{a\}, \{a, b\}\}$, then the smallest topology containing $\mathscr{T}_1$ and $\mathscr{T}_2$ is:

[A] Discrete topology
[B] $\{\Phi, X, \{a\}, \{b\}, \{b, c\}, \{a, b\}\}$
[C] $\{\Phi, X, \{a\}, \{b\}, \{a, c\}, \{a, b\}\}$
[D] $\{\Phi, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$

Q.10. Let $\mathcal{T} = \{(X, \tau): \tau$ is a topology on $X\}$. The relation "~" defined on $\mathcal{T}$ as $X \sim Y$ if and only if X is homeomorphic to Y, then "~" is:

[A] Equivalence relation
[B] Partial order relation
[C] Total order relation
[D] Symmetric but not transitive

SET-Y

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Ans.										

Q.1. Let $X = \{a, b, c\}$, let $\mathcal{T}_1 = \{\Phi, X, \{a\}, \{b, c\}\}$ and $\mathcal{T}_2 = \{\Phi, X, \{a\}, \{a, b\}\}$, then the smallest topology containing $\mathcal{T}_1$ and $\mathcal{T}_2$ is:

[A] $\{\Phi, X, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, b\}\}$
[B] $\{\Phi, X, \{a\}, \{b\}, \{b, c\}, \{a, b\}\}$
[C] $\{\Phi, X, \{a\}, \{b\}, \{a, c\}, \{a, b\}\}$
[D] Discrete topology

Q.2. The subspace $\{0, 1\}$ in $\mathbb{R}$ with usual metric is:

[A] connected **[B]** compact **[C]** clopen **[D]** open

Q.3. A subset of $\mathbb{R}$ which is both compact and connected is:

[A] A closed interval **[B]** An open interval **[C]** A bounded interval **[D]** A finite set

Q.4. If each space X_α is a Hausdorff space, then $\prod_{\alpha\in\wedge} X_\alpha$ is a Hausdorff space in:

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[A] only c **[B]** a but not at c **[C]** both a and b **[D]** only d.

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[C] $\{x \in X: \delta(x, A) > 0\} = A$ **[D]** $\{x \in X: \delta(x, A) > 0\} \subset A$

Q.7. Let $\mathscr{T}_c$ be the co-finite topology on infinite set X:

[A] $(X, \mathscr{T}_c)$ is discrete space **[B]** $(X, \mathscr{T}_c)$ is T_1-space
[C] $(X, \mathscr{T}_c)$ is indiscrete space **[D]** $(X, \mathscr{T}_c)$ is T_2-space

Q.8. For $A \subseteq \mathbb{R}^2$, let $\mathscr{T}$ be the subspace topology on $X = \mathbb{R}^2 \sim A$ then:

[A] A is countable $\Rightarrow (X, \mathscr{T})$ is connected. **[B]** $(X, \mathscr{T})$ is connected $\Rightarrow A$ is finite.
[C] A is unbounded $\Rightarrow (X, \mathscr{T})$ is compact. **[D]** A is open $\Rightarrow (X, \mathscr{T})$ is compact.

Q.9. Let $\mathcal{T} = \{(X, \tau)$**:** τ is a topology on $X\}$. The relation "~" defined on $\mathcal{T}$ as $X \sim Y$ if and only if X is homeomorphic to Y, then "~" is:

[A] Symmetric but not transitive **[B]** Partial order relation
[C] Total order relation **[D]** Equivalence relation

Q.10. Which of the following is falls:

[A] Every metric space is compact Hausdorff space. **[B]** Every metric space is normal space.
[C] Every metric space is regular space. **[D]** Every metric space is Frechet space.

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2022-2023

MATH F311 (Introduction to Topology)

Comprehensive Exam-Part B (Closed Book)

Max Marks: 50 **Time: 120 Min.**

Q.1. Define *topologist sine curve*. Prove or disprove that union of *connected subsets* of a topological space X is *connected.* [10]

Q.2. Define *Sorgenfrey Plane*. Prove or disprove that *compact Hausdorff* space is *normal*. [10]

Q.3. Define limit point compact. Prove or disprove that a topological space is *limit point compact* if and only if it is *compact*. [10]

Q.4. Define Lindelöff space. Prove or disprove that $\mathbb{R}_l$ is *second countable*. [10]

Q.5. Define *finitely short family*. Prove or disprove that a topological space X is *compact* if and only if each *finitely short* family of open sets in X is *short*. [10]