**BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI**
**First Semester 2016 – 17**
**MATH F 312 : Ordinary Differential Equations**
**Mid Semester Test (Closed Book)**

**Duration : 90 mins** **Date : 08/10/2016** **MM : 70**

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Q1. Express the following system of coupled second order scaler differential equations in the vector-matrix form:

$$u'' - u = v'e^t,$$
$$v'' - v = u'e^{-t}.$$ [8]

Q2. State and prove Grownwall-Reid-Bellman Inequality. [12]

Q3. Find all continuous (but not necessarily differentiable) functions $u(t)$ such that

$$u^2(t) \leq \int_0^t u(s)ds\,, \quad t \geq 0.$$ [10]

Q4. Show that all the solutions of the differential equation

$$u'' + \left(1 + \frac{2}{t(t^2+1)}\right)u = 0 \quad for\ \ t > 0$$

are bounded on $(0, \infty)$. [10]

Q5. Find a fundamental matrix $\Phi(t)$ of the following system of differential equations:

$$x_1' = x_1 + 3x_2,$$
$$x_2' = x_1 - x_2.$$ [10]

Q6. Determine the stability/instability/asymptotic stability of the zero solution of the system of differential equations

$$x_1' = 2x_1 - 4x_2,$$
$$x_2' = 7x_1 - 9x_2.$$ [10]

Q7. Let all the solutions of the vector differential equation

$$x' = A(t)x,$$

be bounded on $[0, \infty)$, where $A(t)$ is $n \times n$ continuopus matrix on $[0, \infty)$ and $x$ is $n$-vector. Then show that all solutions of the above differential equation are stable. [10]

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