Birla Institute of Technology & Science, Pilani

Mid Semester Exam (Closed Book) I[st] Semester 2022 – 2023
Course Name : Numerical Analysis (MATH F313) Date: 3rd November 2022

Max. Time: 90 Minutes Max. Marks: 70

Note: Use four significant digits with rounding wherever not mentioned. **Start answering** each question on a **fresh page**.

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1. Solutions of $ax^2+bx+c=0$ ($a\neq 0$) are $x=\left(\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}\right)$. Using this formula, find the smallest root of the equation $0.2x^2-47.913x+6.0003=0$ as accurate as possible. **[4]**

2. To find $\sqrt{17}$ by fixed point method, the following iteration function is proposed: $g(x)=\dfrac{\alpha}{x}+\beta x+\gamma x^3$. Find α,β and γ so that the method converge cubically**. [6]**

3. Find the upper bound of the error in interpolating the values of $f(x)=e^{(-2x^2)}$ at x = 1/3 based on the values at x = 0 and 1. **[6]**

4. Gauss-Elimination (G-E) with scaled partial pivoting is performed on a 3x3 matrix A. After **One- Step** of G-E, the working matrix with scaling factors 7, 4 and 7 respectively, pivoting vector P = (2, 1, 3) and multipliers obtained is as:

$$\begin{bmatrix} 0.2858 & -0.7144 & 1.214 \\ 0.5000 & 1.000 & -0.7500 \\ -0.8572 & 1.857 & -0.3572 \end{bmatrix}$$

   Find the second column of A^{-1} (inverse of A) using the forward and backward substitutions. Hence, find the value of determinant of A. **[12]**

5. Using Newton's method, reduce the nonlinear system:

$$\begin{aligned} 8y-\cos^2(z-y) &= 1 \\ 10x+\sin(x+y) &= 1 \\ 12z+\sin z &= 1 \end{aligned}$$

   to a system of linear equations in h_1, h_2 and h_3 to obtain the solution: $x_1=0.1+h_1$, $y_1=0.25+h_2$ and $z_1=0.0833+h_3$

Hence, perform one iteration of Gauss-Seidel method to find the solution of resulting system in h_1, h_2 and h_3 with initial vector $(1,1,1)^T$ so that the iteration scheme converges to true solution. **[12]**

6. Using the definition of divided difference, complete the following divided difference table:

xi	*fi*	*f*[,]	*f*[,,]	*f*[,,,]	*f*[,,,,]
-1	2				
		-1			
1	…..…		……		
		1		2	
1	…..…		……..		4
		…….		…..	
1	……..		…….		
		…….			
1	……				

Hence, using the above table, find

(a) The values of $f''(1)$ and $f'''(1)$.

(b) The interpolating polynomial ***P(x)*** (in simplified form of degree as high as possible) which interpolate f at above points. **[12]**

7. Derive Newton's backward interpolating formula to approximate a function using Newton's divided difference formula. **[8]**

8. Let A$\boldsymbol{x} = b$ be written as $\boldsymbol{x} = B\,\boldsymbol{x} + c$, with some norm of B, $\|B\| < 1$, then prove that $\boldsymbol{x} = B\,\boldsymbol{x} + c$ has a unique solution $\boldsymbol{R}$. Further, the sequence generated by $\boldsymbol{x}^{(m+1)} = B\,\boldsymbol{x}^{(m)} + c$, $m = 0,1,2,3\ldots$, starting with some initial guess $\boldsymbol{x}^{(0)}$, will converge to the exact solution $\boldsymbol{R}$. **[10]**